Some Properties of Linear Vector Fields on Coset Manifold Spaces

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Abstract

Geometric control theory, a branch of applied mathematics, studies the fundamental principles, theories, and challenges involved in the analysis and design of control systems via differential geometric or topological approach. With this in mind, let us consider the following control system on a smooth(i.e., C^∞) manifold, which has become quite popular in recent years: Assume that M is a finite-dimensional smooth manifold, and $\mathbb{R}^m$ is the m-dimensional Euclidean space. Given a compact convex subset, $\Omega \subset \mathbb{R}^m$ satisfying $0 \in \text{int } \Omega$, we mean by a control-affine system evolving on M the following (parametrized) family of ordinary differential equations

$$\Sigma_M : \quad \dot{x}(t) = f_0(x(t)) + \sum_{j=1}^{m} \omega_j(t) f_j(x(t)), \quad \omega \in \mathcal{U},$$

where $f_0, f_1, \ldots, f_m$ are smooth vector fields defined on M and the control parameter $\omega = (\omega_1, \ldots, \omega_m)$ belongs to the set $\mathcal{U}$ of the piecewise constant functions such that $\omega(t) \in \Omega$.

In the light of recent results by P. Jouan, we investigate how control-affine systems on manifolds can be related to linear systems on Lie groups or coset manifold spaces. Our focus is on determining the conditions under which vector fields on these systems project onto a coset space. Here, using a differential geometric and Lie theoretic approach, we present a result for linear control systems on a coset manifold space of the 3D Heisenberg Lie group.

Keywords: Coset manifolds, Linear vector fields, Heisenberg group
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