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DYNAMIC (HARMONIC) INTERFACIAL STRESS FIELD IN A HALF-PLANE COVERED WITH A PRESTRETCHED SOFT LAYER

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A half-plane covered with a prestretched layer is considered under the action of a periodic dynamic (harmonic) lineal load applied to the free surface of the layer. Within the framework of a piecewise homegeneous body model, with the use of equations of the three-dimensional linearized theory of elastic waves in initially stressed bodies, the problem of stress state is formulated. It is assumed that the materials of the layer and half-plane are linearly elastic, homogeneous, and isotropic, and a plane strain state is considered. The corresponding boundary-value problems are solved analyticaly by employing the exponential Fourier transformations. Numerical results are obtained in the case where the elastic modulus of the half-plane material is greater than that of the layer material. It is established that, because of softening of the layer material, the stresses on the interplane increase mainly in the vicinity of the acting force and this increase has a local character. Moreover, it is established that the prestretching of the cover layer decreases the absolute values of these stresses.

1. Introduction

Intensive inquiries into dynamic problems for elastic bodies with initial stresses, which have a wide range of applications in almost all areas of natural sciences and engineering, such as the mechanics of composites, seismology, biomechanics, etc., started around the second half of the twentieth century. Since that time, numerous theoretical and experimental investigations have been carried out in this field. Except for [1-3] and some other studies, almost all these investigations refer to the influence of initial stresses on the speed and dispersion of various types of waves. Systematic surveys and analyses of the results obtained before 1986 were made in the monographs [4, 5]. The reviews of more recent researches are given in [6-8]. It follows from the analyses that a considerable part of the results obtained refer to layered composite materials with initial stresses. However, the research in the influence of initial stresses on the dynamic stress state in layered media have also a great theoretical and practical significance. The related static problem was examined in [9, 10].

In [1, 2], some attempts were made in this field, and the Lamb problem was considered for the case with a harmonic lineal load applied perpendicularly to the free layer surface. The initial stress state was determined there within the framework of the linear theory of elasticity, and particular investigations were performed for cases where the material of the stratified layer

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was more rigid than the material of the half-plane. In the present paper, the investigations [1, 2] are developed for the case where the elastic modulus of the prestretched cover layer is smaller than that of the half-plane. Note that this situation is more complicated than that considered in [1, 2] and is suitable for considering the propagation of a generalized Rayleigh wave in a covered half-plane [6, 7, 11].

The study is carried out within the framework of a piecewise homogeneous body model with the use of the three-dimensional linearized theory of elastic waves in initially stressed (or prestressed) bodies. Numerical results for the normal and shear stresses acting on the interface plane are obtained, and the influence of problem parameters and the initial stresses are analyzed.

2. Problem Formulation and Solution Method

Consider a half-plane covered with a layer to the free surface of which a harmonic normal lineal load is applied. The cover layer is associated with Lagrangian coordinates $Ox_1x_2x_3$, which in the natural state coincide with Cartesian coordinates. Note that the cover layer and the half-plane occupy the regions $\{-\infty \le x_1 \le \infty, -h \le x_2 \le +h, -\infty \le x_3 \le \infty\}$ and $\{-\infty \le x_1 \le +\infty, -\infty \le x_2 \le -h, -\infty \le x_3 \le \infty\}$, respectively.

In what follows, the quantities related to the layer and the half-plane are denoted by superscripts (1) and (2), respectively, whereas those related to the initial stresses are denoted by the superscripts (m),0, where $m = 1, 2$.

Consider the case where the initial stresses in the layer and the half-plane are

$$\sigma_{11}^{(m),0} = \text{const}_m \ne 0,\ m = 1, 2; \quad \sigma_{ij}^{(m),0} = 0 \quad \text{at} \quad i = j \ne 1. \tag{1}$$

The linearly elastic material of the layer and the half-plane is assumed homogeneous and isotropic. Since $\sigma_{11}^{(m),0}$ is the only nonzero component of the initial stress tensor both in the layer and in the half-plane, $\sigma_{11}^{(m),0}$ is denoted for short as $\sigma_0^{(m)}$. It can be assumed that the plane deformation state in the Ox_1x_2 plane prevails. According to [4, 5], for the case considered, we have

$$\frac{\partial \sigma_{ij}^{(m)}}{\partial x_j} + \sigma_0^{(m)} \frac{\partial^2 u_i^{(m)}}{\partial x_1^2} = \rho_0^{(m)} \frac{\partial^2 u_i^{(m)}}{\partial t^2}, \quad i, j = 1, 2;\ m = 1, 2. \tag{2}$$

Here, $\rho_0^{(m)}$ is the density of an mth material in the natural state. The other designations in (2) are conventional.

For an isotropic compressible material, we can write

$$\sigma_{ij}^{(m)} = \lambda^{(m)} \theta^{(m)} \delta_{ij} + 2\mu^{(m)} \varepsilon_{ij}, \quad \theta^{(m)} = \varepsilon_{11}^{(m)} + \varepsilon_{22}^{(m)}, \tag{3}$$

where $\lambda^{(m)}$ and $\mu^{(m)}$ are the Lamé constants and

$$\varepsilon_{ij}^{(m)} = \frac{1}{2}\left(\frac{\partial u_i^{(m)}}{\partial x_j} + \frac{\partial u_j^{(m)}}{\partial x_i} \right). \tag{4}$$

We assume the following complete contact conditions between the layer and the half-plane:

$$u_i^{(1)}\Big|_{x_2=-h} = u_i^{(2)}\Big|_{x_2=-h}, \quad \sigma_{i2}^{(1)}\Big|_{x_2=-h} = \sigma_{i2}^{(2)}\Big|_{x_2=-h}, \quad i = 1, 2. \tag{5}$$

In the free face plane of the cover layer

$$\sigma_{12}^{(1)}\Big|_{x_2=0} = 0, \quad \sigma_{22}^{(1)}\Big|_{x_2=0} = -P\delta(x_1)e^{i\omega t}. \tag{6}$$

In addition, we also assume that at there are no reflections at $x_2 \to -\infty$, which means that all waves propagate in the negative x_2 direction. In other words,

$$|u_i^{(2)}|, |\sigma_{ij}^{(2)}| < M = \text{const} \quad \text{at} \quad x_2 \to -\infty. \tag{7}$$

Thus, the problem under study is reduced to Eqs (2)-(4) with contact conditions (5) and boundary conditions (6), (7).

Substituting (3) and (4) in (2), we obtain the following linearized equations of motion in terms of displacement perturbations:

$$\begin{aligned}
&(\lambda^{(m)} + 2\mu^{(m)} + \sigma_0^{(m)})\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \mu^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial x_2^2} \\
&+ (\lambda^{(m)} + \mu^{(m)})\frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial x_2} = \rho_0^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial t^2}, \\
&(\lambda^{(m)} + \mu^{(m)})\frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial x_2} + (\mu^{(m)} + \sigma_0^{(m)})\frac{\partial^2 u_2^{(m)}}{\partial x_1^2} \\
&+ (\lambda^{(m)} + 2\mu^{(m)})\frac{\partial^2 u_2^{(m)}}{\partial x_2^2} = \rho_0^{(m)}\frac{\partial^2 u_2^{(m)}}{\partial t^2}.
\end{aligned} \tag{8}$$

As in [1, 2], we represent the displacements $u_1^{(m)}$ and $u_2^{(m)}$ in terms of potentials $\varphi^{(m)}$ and $\psi^{(m)}$ as

$$u_1^{(m)} = \frac{\partial \varphi^{(m)}}{\partial x_1} + \frac{\partial \psi^{(m)}}{\partial x_2}, \quad u_2^{(m)} = \frac{\partial \varphi^{(m)}}{\partial x_2} - \frac{\partial \psi^{(m)}}{\partial x_1}. \tag{9}$$

By direct verification, it can be shown that the potentials must satisfy the equations

$$\begin{aligned}
&\nabla^2 \varphi^{(m)} + \frac{\sigma_0^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}}\frac{\partial^2 \varphi^{(m)}}{\partial x_1^2} = \frac{1}{(c_1^{(m)})^2}\frac{\partial^2 \varphi^{(m)}}{\partial t^2}, \\
&\nabla^2 \psi^{(m)} + \frac{\sigma_0^{(m)}}{\mu^{(m)}}\frac{\partial^2 \psi^{(m)}}{\partial x_1^2} = \frac{1}{(c_2^{(m)})^2}\frac{\partial^2 \psi^{(m)}}{\partial t^2},
\end{aligned} \tag{10}$$

where

$$c_1^{(m)} = \sqrt{\frac{\lambda^{(m)} + 2\mu^{(m)}}{\rho_0^{(m)}}}, \quad c_2^{(m)} = \sqrt{\frac{\mu^{(m)}}{\rho_0^{(m)}}}$$

are the speeds of dilatation and distortion waves, respectively. At $\sigma_0^{(m)} = 0$, Eqs. (10) coincide with those in the classical linear theory of elastodynamics [11].

Let us consider the solution of equations (10). Since the lineal load is harmonic in time, only the stationary case is to be considered. All dependent variables are also harmonic and can be represented as

$$\{u_n^{(m)}, \sigma_{nj}^{(m)}, \varepsilon_{nj}^{(m)}\}(x_1, x_2, t) = \{\overline{u}_n^{(m)}, \overline{\sigma}_{nj}^{(m)}, \overline{\varepsilon}_{nj}^{(m)}\}(x_1, x_2) e^{i\omega t}, \tag{11}$$

where the superposed bar denotes the amplitude of the corresponding quantity. Hereafter these bars will be omitted.

Using Eq. (11) in (2)-(10) and replacing $\dfrac{\partial^2 u_i^{(m)}}{\partial t^2}$ with $-\omega^2 u_i^{(m)}$, we obtain the same equations and conditions for the amplitudes of sought-for quantities. Introducing the dimensionless coordinates $x_i \to x_i / h$ and the dimensionless frequency

$$\Omega = \omega h \Big/ c_2^{(1)},$$

we arrive at the following equations for the potentials $\varphi^{(m)}$ and $\psi^{(m)}$:

$$\nabla^2 \varphi^{(m)} + \frac{\sigma_0^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}} \frac{\partial^2 \varphi^{(m)}}{\partial x_1^2} + \frac{(c_2^{(1)})^2}{(c_1^{(m)})^2} \Omega^2 \varphi^{(m)} = 0,$$

$$\nabla^2 \psi^{(m)} + \frac{\sigma_0^{(m)}}{\mu^{(m)}} \frac{\partial^2 \psi^{(m)}}{\partial x_1^2} + \frac{(c_2^{(1)})^2}{(c_2^{(m)})^2} \Omega^2 \psi^{(m)} = 0. \tag{12}$$

To solve Eqs. (12), we employ the exponential Fourier transformation

$$f_F(sx_2) = \int_{-\infty}^{+\infty} f(x_1, x_2) e^{isx_1} dx_1. \tag{13}$$

After applying (13) to Eqs. (12), the functions $\varphi_F^{(m)}$ and $\psi_F^{(m)}$ become

$$\begin{aligned} \overline{\varphi}_F^{(1)} &= A_1^{(1)}(s) e^{\gamma_1^{(1)} x_2} + A_2^{(1)}(s) e^{-\gamma_1^{(1)} x_2}, \quad \overline{\varphi}_F^{(2)} = A_1^{(2)}(s) e^{\gamma_1^{(2)} x_2}, \\ \overline{\psi}_F^{(1)} &= B_1^{(1)}(s) e^{\gamma_2^{(1)} x_2} + B_2^{(1)}(s) e^{-\gamma_2^{(1)} x_2}, \quad \overline{\psi}_F^{(2)} = B_1^{(2)}(s) e^{\gamma_2^{(2)} x_2}, \end{aligned} \tag{14}$$

where

$$(\gamma_1^{(m)})^2 = s^2 \left(1 + \frac{\sigma_0^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}} \right) - \frac{(c_2^{(1)})^2}{(c_1^{(m)})^2} \Omega^2,$$

$$(\gamma_2^{(m)})^2 = s^2 \left(1 + \frac{\sigma_0^{(m)}}{\mu^{(m)}} \right) - \frac{(c_2^{(1)})^2}{(c_2^{(m)})^2} \Omega^2.$$

The unknowns $A_1^{(1)}$, $A_2^{(1)}$, $A_1^{(2)}$, $B_1^{(1)}$, $B_2^{(1)}$, and $B_1^{(2)}$ in Eqs. (14) are determined from contact conditions (5) and boundary conditions (6), (7). By a usual mathematical procedure, we also find the expressions for $u_{nF}^{(m)}$, $\varepsilon_{njF}^{(m)}$, and $\sigma_{njF}^{(m)}$ (n, j, m= 1, 2), from which the original of the required quantities is represented as

$$\{u_n^{(m)}, \sigma_{nj}^{(m)}, \varepsilon_{nj}^{(m)}\}(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \{u_{nF}^{(m)}, \varepsilon_{njF}^{(m)}, \sigma_{njF}^{(m)}\}(sx_2) e^{-isx_1} ds.$$

The expressions of $u_{nF}^{(m)}$, $\varepsilon_{njF}^{(m)}$, and $\sigma_{njF}^{(m)}$ are cumbersome and therefore are omitted here.

For determining the stresses acting on the interplane, we calculate the integrals

$$\sigma_{n2}^{(m)}(x_1,-1)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\sigma_{n2F}^{(m)}(-s)e^{-isx_1}\,ds.$$

numerically.

In view of symmetry of the problem, the integrals become

$$\sigma_{22}^{(m)}(x_1,-1)=\frac{1}{\pi}\int\limits_{0}^{+\infty}\sigma_{22F}^{(m)}(-s)\cos(sx_1)ds,$$

$$\sigma_{12}^{(m)}(x_1,-1)=\frac{-i}{\pi}\int\limits_{0}^{+\infty}\sigma_{12F}^{(m)}(-s)\sin(sx_1)ds. \tag{15}$$

Note that the expression of $\sigma_{12F}^{(m)}$ has the form $i\left|\sigma_{12F}^{(m)}\right|$, therefore the values of the second integral in (15) are real.

In calculating Eqs. (15), the improper integrals are replaced by corresponding definite ones: $\int\limits_0^{\infty}(\cdot)\,ds\approx\int\limits_0^{S_*}(\cdot)\,ds$. The values of S_* are determined from a convergence criterion of the integrals.

Let us find the singular points of Eqs. (15). Since the unknowns in (14) can be expressed as

$$(A_1^{(1)}(s),A_2^{(1)}(s),\ldots,B_2^{(2)}(s))$$

$$=\frac{1}{\det\left\|\alpha_{ij}(s)\right\|}\left(\det\left\|\beta_{ij}^{A_1^{(1)}(s)}\right\|,\det\left\|\beta_{ij}^{A_2^{(1)}(s)}\right\|,\ldots,\det\left\|\beta_{ij}^{B_2^{(2)}(s)}\right\|\right),$$

the singular points coincide with roots of the equation

$$\det\left\|\alpha_{ij}(s)\right\|=0\quad(i,\,j=1,2,3,4,5,6) \tag{16}$$

with respect to s, where α_{ij} are the coefficients of unknowns in the system of algebraic equations obtained from contact (5) and boundary (6) conditions. Note that the expressions for $\left\|\beta_{ij}^{A_1^{(1)}(s)}\right\|$, $\left\|\beta_{ij}^{A_2^{(1)}(s)}\right\|$, ..., and $\left\|\beta_{ij}^{B_2^{(2)}(s)}\right\|$ are found from $\left\|\alpha_{ij}(s)\right\|$ by replacing the corresponding column with the right side of the system of algebraic equations mentioned.

A numerical analysis shows that the order of roots of Eq. (16) with respect to s is one. Therefore, the order of all singular points of Eqs. (15) is also one, and Eq. (16) can be solved by the well-known bisection method.

Let us denote the roots of Eq. (16) by

$$s_1<s_2<\ldots<s_k<\ldots<s_N. \tag{17}$$

It can be shown that, among these roots, there are always such s for which $\gamma_1^{(m)}(s)=0$ and $\gamma_2^{(m)}(s)=0$. Moreover, the number of roots N depends mainly on the dimensionless frequency Ω and the ratio $E^{(1)}/E^{(2)}$. The values of $E^{(1)}/E^{(2)}$ considered in the present paper, at $\nu^{(1)}=\nu^{(2)}$ ($\nu^{(m)}$ are the Poisson ratios) and $\rho_0^{(1)}=\rho_0^{(2)}$, correspond to the case where $c_2^{(1)}<c_1^{(1)}<c_2^{(2)}<c_1^{(2)}$, which, according to [6, 7, 11], is suitable for propagation of generalized Rayleigh waves in the covered half-plane. Therefore, the root s_k in (15) for which $c_1^{(1)}/c_2^{(1)}<\Omega/s_k<c_2^{(2)}/c_2^{(1)}$ can be taken as the dimensionless wavenumber of the corresponding generalized Rayleigh wave.

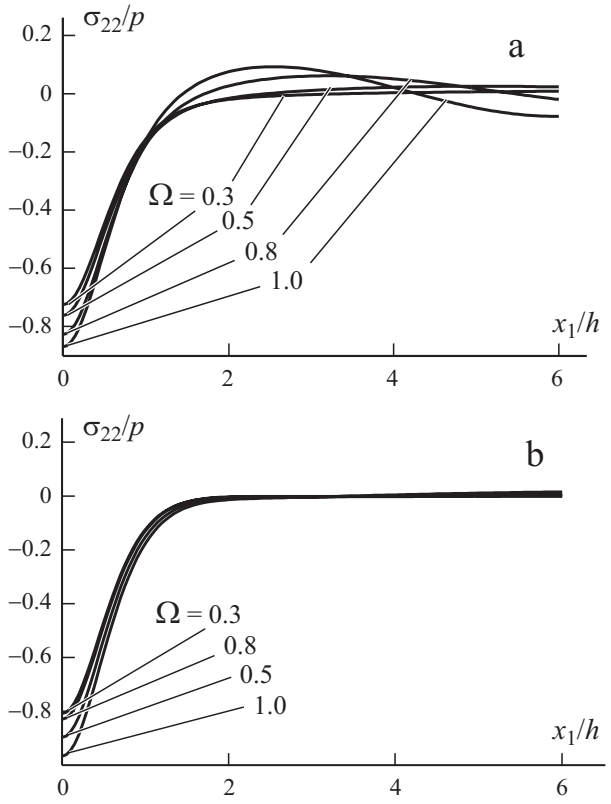


Fig. 1. The stress σ_{22} as a function of x_1/h for various dimensionless frequencies Ω at $e = 0.5$ (a) and 0.05 (b); $\eta_1 = 0$.

It should be noted that, in [1, 2], the case $c_1^{(1)} > c_2^{(1)} > c_1^{(2)} > c_2^{(2)}$ was considered, for which generalized Rayleigh waves are not suitable.

After determining roots (17), the integration interval $[0, S_*]$ in (15) breaks down into subintervals:

$$\int_0^{S_*} (\cdot)\, ds = \left[\int_0^{s_1-\varepsilon} (\cdot)\, ds + \int_{s_1+\varepsilon}^{s_2-\varepsilon} (\cdot)\, ds + \ldots + \int_{s_N+\varepsilon}^{S_*} (\cdot)\, ds \right].$$

Thus, integral (15) is taken in the sense of Cauchy principal value, where ε is a small quantity, which is determined from the convergence requirement for integral (15). Further, each of the integration intervals $[s_k + \varepsilon, s_{k+1} - \varepsilon]$ is also divided into a certain number of shorter intervals and, for each of them, the Gauss integration algorithm is employed. All these procedures are performed automatically by using computer programs written in MATLAB.

3. Numerical Results and Discussion

We assume that $\sigma_0^{(1)} > 0$, $\sigma_0^{(2)} = 0$, $\rho_0^{(1)} = \rho_0^{(2)}$, $E^{(1)}/E^{(2)} < 1$, and $\nu^{(1)} = \nu^{(2)} = 0.25$, where $\nu^{(1)}$ ($\nu^{(2)}$) is the Poisson ratio of the layer (half-plane) material, and introduce the parameter $\eta_1 = \sigma_0^{(1)}/\mu^{(1)}$ and the designations

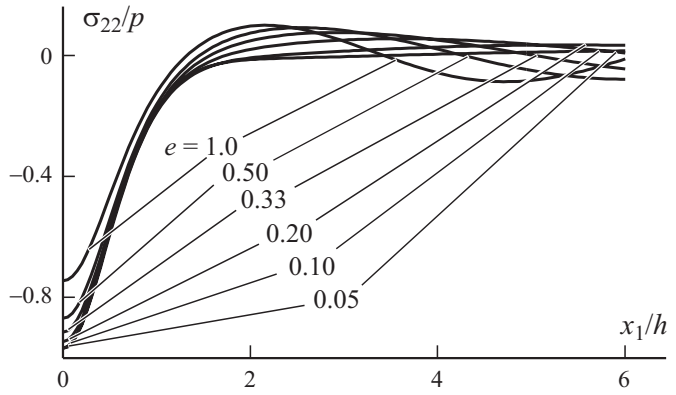


Fig. 2. Influence of the parameter e on the distribution of σ_{22} with respect to x_1/h at $\Omega = 1$ and $\eta_1 = 0$.

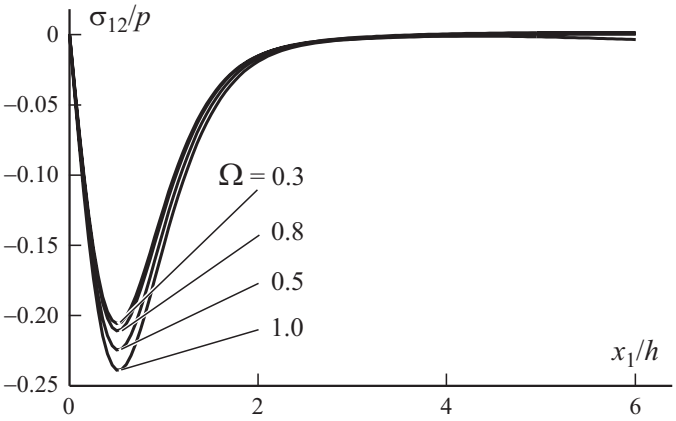


Fig. 3. Influence of the dimensionless frequency Ω on the distribution of σ_{12} with respect to x_1/h at $e = 0.05$ and $\eta_1 = 0$.

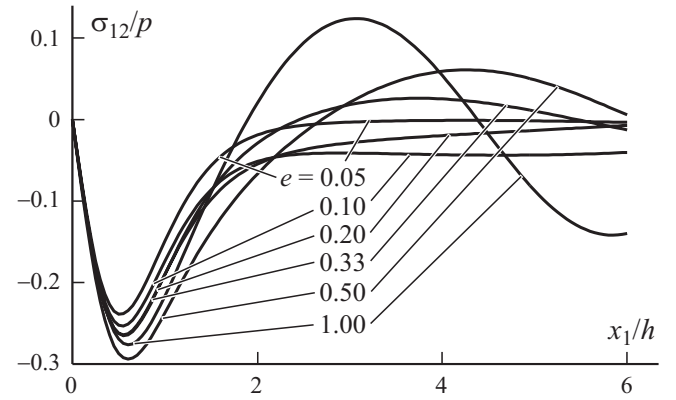


Fig. 4. Influence of the parameter e on the distribution of σ_{12} with respect to x_1/h at $\Omega = 1$ and $\eta_1 = 0$.

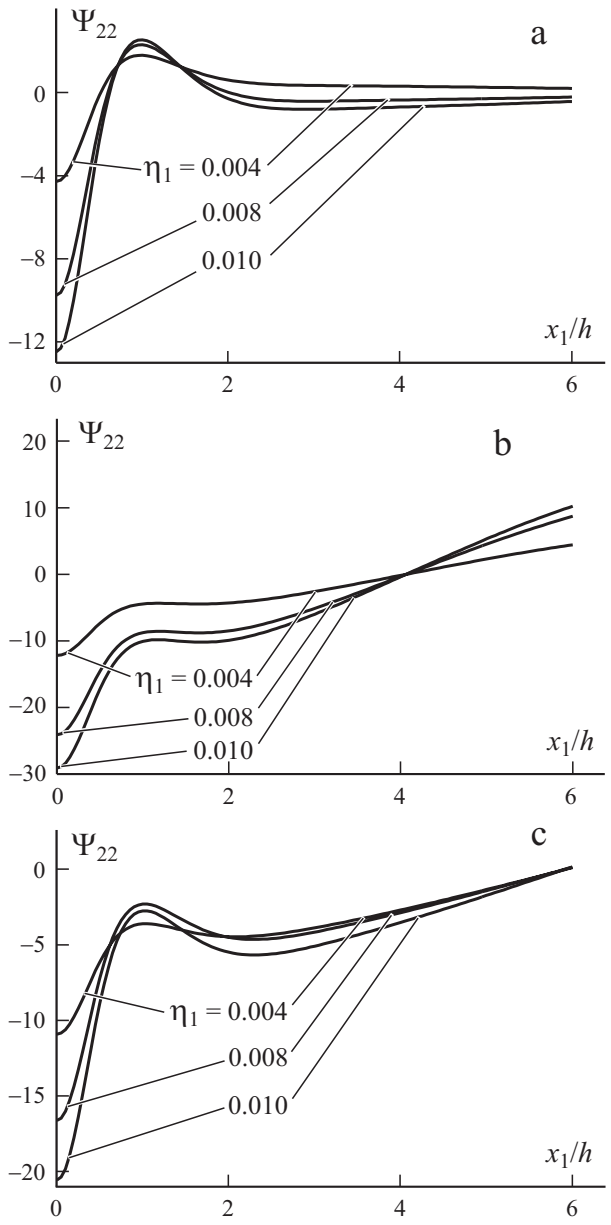


Fig. 5. Distribution of Ψ_{22} with respect to x_1/h for various initial stretchings (i.e., for various η_1): $e = 0.1, \Omega = 0.5$ (a); $e = 0.1, \Omega = 1$ (b); $e = 0.05, \Omega = 1$ (c).

$\sigma_{22}^{(1)}\big|_{x_2=-1} = \sigma_{22}^{(2)}\big|_{x_2=-1} = \sigma_{22}$ and $\sigma_{12}^{(1)}\big|_{x_2=-1} = \sigma_{12}^{(2)}\big|_{x_2=-1} = \sigma_{12}$. The parameter η_1 will be utilized to investigate the influence of prestretching the cover layer on the distribution of the stresses σ_{22} and σ_{12}. This investigation will be conducted with account of the following two facts:

1. The values of σ_{22} and σ_{12} obtained for $\Omega = 0.00, \ldots, 0.10$ are hardly discernible. For $E^{(1)}/E^{(2)} = 1$, these values coincide with the solution of the corresponding static problem given, for example, in [12] and elsewhere.

2. The numerical results obtained for the case of $E^{(1)}/E^{(2)} > 1$ coincide with those found in [1, 2].

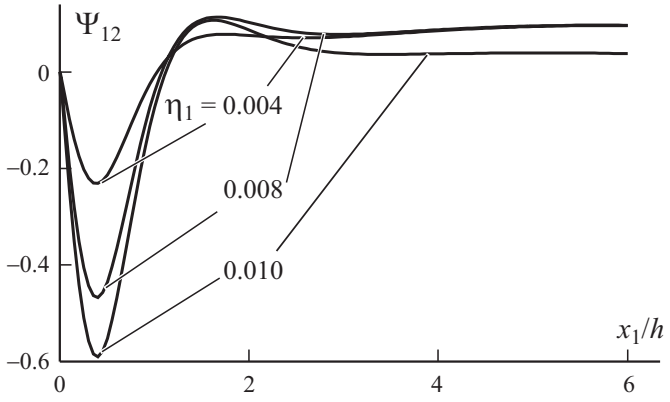


Fig. 6. Distribution of Ψ_{12} with respect to x_1/h for various η_1 at $\Omega = 1$ and $e = 0.05$.

These facts guarantee the validity of the algorithms and programs used in the present investigations.

Fig. 1 shows the distribution of σ_{22}/p with respect to x_1 for various Ω at $e = E^{(1)}/E^{(2)} = 0.5$ (Fig. 1a) and 0.05 (Fig. 1b). It I seen that the maximal absolute values of σ_{22}/p increase with Ω. Fig. 2 shows the same distribution for various e at $\Omega =$ 1.0. As is seen, the corresponding values of σ_{22}/p approach each other asymptotically with decreasing e. Consequently, the cases where $e \le 0.05$ can be considered as covered absolutely rigid half-planes. Moreover, it follows from the results given in Fig. 2 that, with decreasing e, the oscillating character of distribution of σ_{22}/p disappears.

Note that similar results are obtained for the distribution of σ_{12}/p with respect to x_1. These results are illustrated in Figs. 3 and 4.

Let us consider the influence of initial stretching of the cover layer on the stresses σ_{22} and σ_{12}. For this purpose we introduce the designation

$$\Psi_{i2} = \left(\sigma_{i2}\big|_{\eta_1=0} - \sigma_{i2}\big|_{\eta_1>0} \right) \cdot 10^3 \Big/ p, \quad i = 1, 2.$$

The graphs for Ψ_{22} and Ψ_{12} as functions of x_1 are given in Figs. 5 and 6. The corresponding problem parameters are shown in the figures.

It follows from these graphs that the initial stretching of the cover layer decreases the maximal absolute value of σ_{22} and σ_{12}. These values decrease monotonically with η_1.

4. Conclusion

In the present paper, the Lamb problem for a half-plane covered with a prestretched soft layer has been investigated within the framework of a piecewise homogeneous body model by employing the three-dimensional linearized theory of elastic waves in initially stressed bodies. According to the numerical results obtained, in particular, we can draw the following conclusions.

For the range of Ω considered (i.e., for $0.3 \le \Omega \le 1.0$), the maximum absolute values of stresses increase with Ω and approach each other asymptotically with decreasing $\le \Omega \le$. The corresponding graphs obtained for $e \le 0.02$ coincide.

In the cases where $e << 1$, the oscillating character of the stress distribution considered, which was observed for $e > 1$ [1, 2], disappears completely. This fact explains why it is easier to apply a layer of some material to the surface of harder ones, to make the upper face smoother, than to cover soft materials with hard ones.

The results obtained for $e = 0.05$ correspond to the case where the layer lies on an absolutely rigid substrate.

In all the cases considered, the influence of prestretching the layer on the stress distribution considered increases monotonically with prestretch level, and the prestretching of the layer decreases the maximum absolute values of the stresses analyzed.

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