Soru: $I=\int \sqrt{(2+x-x^2)^5}\,dx = ?$

$$2+x-x^2 = -(x^2-x-2) = -\left[\left(x-\frac{1}{2}\right)^2-\frac{9}{4}\right]$$
$$= \frac{9}{4}-\left(x-\frac{1}{2}\right)^2$$

$$I=\int \sqrt{\left[\frac{9}{4}-\left(x-\frac{1}{2}\right)^2\right]^5}\,dx$$

$$x-\frac{1}{2}=\frac{3}{2}\sin t \;;\; dx=\frac{3}{2}\cos t\,dt$$

$$I=\int \frac{3^5}{2^5}\cdot\cos^5 t\cdot\frac{3}{2}\cos t\,dt = \frac{3^6}{2^6}\cdot\int \cos^6 t\,dt$$

$$I=\frac{729}{64}\int\left(\frac{1+\cos 2t}{2}\right)^3 dt = \frac{729}{512}\int\left(1+3\cos 2t+3\cos^2 2t+\cos^3 2t\right)dt$$

$$I=\frac{729}{512}\left[t+\frac{3}{2}\sin 2t+3\cdot\int\frac{1+\cos 4t}{2}\,dt+\int(1-\sin^2 2t)\cos 2t\,dt\right]$$

$$I=\frac{729}{512}\left[t+\frac{3}{2}\sin 2t+\frac{3}{2}\left(t+\frac{1}{4}\sin 4t\right)+\frac{1}{2}\left(\sin 2t-\frac{1}{3}\sin^3 2t\right)\right]+C$$

$\sin t=\dfrac{2x-1}{3}$, $\quad \sin 2t = 2\sin t\cos t = 2\cdot\left(\dfrac{2x-1}{3}\right)\cdot\sqrt{1-\left(\dfrac{2x-1}{3}\right)^2}$

$\cos t=\sqrt{1-\left(\dfrac{2x-1}{3}\right)^2}$, $\quad \sin 4t = 2\sin 2t\cos 2t$

$\cos 2t=\cos^2 t-\sin^2 t = 1-\left(\dfrac{2x-1}{3}\right)^2-\left(\dfrac{2x-1}{3}\right)^2$

yerine yazılarak sonuç bulunur.