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Fate of the Universe, Age of the Universe, Dark Matter, and the Decaying Vacuum Energy

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ABSTRACT

It is shown that in cosmological models based on a vacuum energy decaying as a^{-2}, where a is the scale factor of the universe, the fate of the universe in regard to whether it will collapse in the future or expand forever is determined not by the curvature constant k but by an effective curvature constant k_{eff}. It is argued that a closed universe with $k = 1$ may expand forever, in other words simulate the expansion dynamics of a flat or an open universe because of the possibility that $k_{eff} = 0$ or -1, respectively. Two such models, in one of which the vacuum does not interact with matter and in another of which it does, are studied. It is shown that the vacuum equation of state $p_{vac} = -\rho_{vac}$ may be realized in a decaying vacuum cosmology provided the vacuum interacts with matter. The optical depths for gravitational lensing as a function of the matter density and other parameters in the models are calculated at a source redshift of 2. The age of the universe is discussed and shown to be compatible with the new *Hipparcos* lower limit of $11Gyr$. The possibility that a time-varying vacuum energy may serve as dark matter is suggested.

Subject headings: Cosmology: theory, dark matter, gravitational lensing

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1. INTRODUCTION

Homogeneous and isotropic cosmological models based on a time-varying cosmological term $\lambda(t) \propto a(t)^{-2}$, with $a(t)$ being the scale factor of the universe, do not suffer from the

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notorious problems of the standard hot big-bang cosmology such as the initial singularity, horizon (causality), entropy, monopole, and cosmological constant problem (Özer & Taha 1986, 1987). However, these are not the only successes of such cosmological models. There are others. In the homogeneous and isotropic cosmological models based on the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2) \right] \tag{1}$$

the universe is closed and its expansion will halt and contraction will start if the curvature constant $k = 1$, the universe is open and will expand forever if $k = -1$, and the universe is flat and will expand forever if $k = 0$. As is well known, these conclusions concerning the expansion of the universe follow from the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_M(t) + \frac{\lambda}{3} - \frac{k}{a^2}, \tag{2}$$

if the (time-independent) cosmological constant λ vanishes. Here $\rho_M(t)$ is the density of relativistic matter (photons and other relativistic particles) in the radiation-dominated era and the density of nonrelativistic matter (mainly luminous and nonluminous baryons and possibly massive particles such as axions) in the matter-dominated era. We have set the speed of light c equal to 1. However, these conclusions can drastically change if the cosmological constant λ is nonzero. Depending on the value of λ, a closed universe may collapse or expand forever for positive λ, an open or flat universe may collapse for negative λ (see, for example, Rindler 1977; Landsberg & Evans 1979; Felten & Isaacman 1986). If, on the other hand, λ is allowed to vary with time according to $\lambda \propto a^{-2}$ not only some of the above features are maintained but the expansion dynamics of a closed universe may be similar to that of a flat or an open universe.

In the standard model (hereafter SM) [2] , the relation between the present value H_0 of Hubble's constant $H = \dot{a}/a$ and the present age t_0 of the universe is given by (Al-Rawaf & Taha 1996)

$$H_0 t_0 = \frac{1}{(1-\Omega_M)} \left[1 - \frac{\Omega_M}{(1-\Omega_M)^{1/2}} sinh^{-1}(\Omega_M^{-1} - 1)^{1/2} \right] , \quad k = -1 \tag{3a}$$

[2]Felten & Isaacman(1986) call the models with $\lambda = 0$ "standard models". However, we follow the general trend in the literature and call the totality of them the "standard model" and refer to each case by its k value (see, for example, Misner, Thorne, & Wheeler 1970; Weinberg 1972.) Models with zero and nonzero pressure, with or without λ in both cases, are called the Friedmann models and the Lemaître models, respectively. The SM with $k = 0$ is called the Einstein-de Sitter model (Felten & Isaacman 1986).

$$= \quad 2/3 \ , \qquad\qquad k = 0 \tag{3b}$$

$$= \quad \frac{1}{(\Omega_M - 1)} \left[\frac{\Omega_M}{(\Omega_M - 1)^{1/2}} sin^{-1}(1 - \Omega_M^{-1})^{1/2} - 1 \right] \ , \ k = 1 \tag{3c}$$

where Ω_M [3] is the present value of the matter density parameter defined as the ratio of the present values of the matter energy density to the critical energy density

$$\Omega_M = \frac{\rho_M}{\rho_c} = \frac{\rho_M}{3H_0^2/8\pi G} \tag{4}$$

At this point let us also define $\rho_\Lambda = \lambda/8\pi G$, the energy density associated with the cosmological constant and $\Omega_\Lambda = \rho_\Lambda/\rho_c$, the fraction of the present critical density contributed today by the cosmological term. Recently, new parallax measurements obtained by the *Hipparcos* satellite have given the age of the globular clusters, hence the age of the universe, as between 11 to 13 Gyr (Reid 1997; Feast & Cathcpole 1997; Schwarzschild 1997). These time scales are considerably lower than the previous estimates of $16 \pm 2Gyr$ (Peebles 1993; Chaboyer, Demarque, & Sarajedini 1996; Sandquist, Bolte, Stetson, &Hesser 1996; Bolte & Hogan 1995) which are in agreement with the SM for $k = -1$ if

$$H_0 = 55 \pm 5 kms^{-1}Mpc^{-1},$$

(Sandage *et al.* 1996; Tammann & Sandage 1996) and in conflict with it if

$$H_0 = 73 \pm 10 kms^{-1}Mpc^{-1},$$

(Freedman, Madore, & Kennicut 1997). Reid (1997) has argued that the *Hipparcos* data reveal a 7% increase in the distances inferred from the previous ground-based data, implying a decrease in H_0. He thus concludes that the recent Freedman *et al.* (1997) value of H_0 is reduced to $H_0 = 68 \pm 9 kms^{-1}Mpc^{-1}$. Sandage, however, points out that their value of $H_0 = 55 \pm 5 kms^{-1}Mpc^{-1}$ is unaltered (Sandage 1997, preprint). We should add in passing that ever since its first determinations by Hubble, the value of H_0 is still one of the most controversial parameters of Astronomy. The values found by different people using different methods continue to disagree. For example, to early March 1996 the lowest value reported is $H_0 = 55^{+8}_{-4} kms^{-1}Mpc^{-1}$ (from surface brightness fluctuations) whereas the highest one is $H_0 = 84 \pm 4 kms^{-1}Mpc^{-1}$ (from luminosity function of planetary nebulae) (Trimble 1996). To our knowledge, the most recent upper bound on H_0 has been determined by the Supernova Cosmology Project. Using the first seven supernovae at $z \geq 0.35$ that have recently been discovered, Kim *et al.* (1997) have measured the Hubble constant to be

[3] We will denote the current values of the densities and the density parameters such as ρ_{M0} and Ω_{M0} by ρ_M and Ω_M. The time-dependence of variable parameters will be denoted explicitly as in $\rho_M(t)$.

$< 83 kms^{-1}Mpc^{-1}$ and $< 78 kms^{-1}Mpc^{-1}$ in a flat universe with $\Omega_M \geq 0$ and $\Omega_M \geq 0.2$, respectively. Writing $H_0 = 100 hkms^{-1}Mpc^{-1}$, in our discussions below we will use h values in the range 0.5 to 0.8. Therefore, in the light of the recently determined moderately high values of H_0 one cannot claim for sure that the age of the universe problem in the SM has disappeared. High values of H_0 force the age to fall below the *Hipparcos* lower limit of $11Gyr$. In Figure 1 we present the ages in the SM for the three values of k. It is seen that for $k = 0$ the ages are between 8 to $10Gyr$ if $h > 0.6$. Now for low Ω_M and h values between 0.5 to 0.6, the predicted ages are greater than $13Gyr$. As Ω_M gets larger the favored h values shift towards smaller ones. The problem continues to exist if $\Omega_M = 0.2$ and $h > 0.75$, $\Omega_M = 0.3$ to 0.5 and $h \geq 0.75$, $\Omega_M = 0.6$ and $h > 0.65$, $\Omega_M = 0.7$ to 0.8 and $h \geq 0.65$, $\Omega_M = 0.9$ to 1.0 and $h > 0.6$.

Long before *Hipparcos*, when the estimated ages of the globular clusters were in the range $16 \pm 2Gyr$ and hence the age problem was starker, an immediate solution to this likely problem was provided in the 1980's by including a (time-independent) cosmological constant λ as in eq.(2) (Peebles 1984; Blome & Priester 1985; Klapdor & Grotz 1986). Another solution to this problem was given by Olson & Jordan (1987) in the framework of a time-varying cosmological constant. They showed that in a flat universe with $k = 0$ ages of the universe old enough to agree with observations (in the 1980's) could be obtained with background energy densities of the form $\rho_b = \rho_{b0}(a_0/a)^b$, where $b \geq 0$.

Still another potential function of a time-varying cosmological constant is that the vacuum energy density associated with it can be interpreted as the density of dark matter. The nature of the dark matter that is supposed to exist around galaxies has been another most debated mysteries of Astronomy and Cosmology. Many relativistic and nonrelativistic particles have been proposed as candidates for dark matter (see the latest reviews by Primack (1996) and Srednicki (1996)).

The purpose of this paper is to study a phenomenological cosmological model based on the vacuum energy density $\rho_\Lambda(t) = C_1\rho_M + C_2\rho_M a_0^2/a^2$ (see eq.(6) below) in the matter dominated era and show that (i) the part of $\rho_\Lambda(t)$ decaying as a^{-2} leads to an effective curvature constant k_{eff}, and it is k_{eff} that governs the fate of the universe, (ii) such a $\rho_\Lambda(t)$ solves the age problem of the universe, and (iii) $\rho_\Lambda(t)$ serves as dark matter. This paper is organized as follows. In section 2 we mention the time-varying cosmological constant models briefly, and introduce in section 3 the model we study in this paper. We then present our calculations under two different assumptions regarding the interaction of the vacuum with matter. Section 4 concludes the paper.

2. THE TIME-VARYING COSMOLOGICAL CONSTANT MODELS

The time-varying cosmological constant models with $\lambda(t) = const.a^{-2}$ were introduced by Özer & Taha (1986, 1987) in an attempt to solve the cosmological problems such as the initial singularity, horizon, entropy, monopole, and cosmological constant problem (see Weinberg (1989) for a review of the cosmological constant problem). The idea of Özer & Taha (1986, 1987) was then extended to include a large variety of varying cosmological constants decaying as a^{-2} (Gasperini 1987; Chen & Wu 1990, 1992; Berman 1991; Özer & Taha 92; Abdel-Rahman 1992; Carvalho, Lima, & Waga 1992; Waga 1993; Arbab & Abdel-Rahman 1994; Matyjasek 1995). In particular, it was shown by Chen & Wu (1990, 1992) that very general arguments from Quantum Cosmology lead to this form for the effective cosmological constant (see below).

3. THE MODEL

Any cosmological model with a cosmological constant is based on the observation that there is an associated energy density $\rho_\Lambda = \lambda/8\pi G$ in terms of which eq.(2) can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_M(t) + \rho_\Lambda(t)\right] - \frac{k}{a^2}. \tag{5}$$

The energy density ρ_Λ is interpreted as the vacuum energy density (see e.g. Weinberg 1989). As we have stated above, in this paper we shall consider models in which the dependence of the vacuum energy on the scale factor a is of the form

$$\rho_\Lambda(t) = C_1\rho_M + C_2\rho_M\frac{a_0^2}{a^2}, \tag{6}$$

where C_1 and C_2 are constants, ρ_M and a_0 are the present values of the matter energy density ρ_M and the scale factor a. The reason for adopting the variable part of $\rho_\Lambda \propto a^{-2}$ is manyfolds. First, postulating that $\rho_\Lambda \propto a^{-n}$ $(n \neq 0)$, it is only for $n = 2$ that an effective curvature constant k_{eff} can be defined. Second, observing that the cosmological constant λ has dimension of inverse lenght squared, the simplest scale factor dependence of λ would be $\propto a^{-2}$. This is the case here because the time-varying cosmological constant that corresponds to eq.(6) has the form

$$\lambda(t) = \lambda_1 + \lambda_2 a(t)^{-2}, \tag{7}$$

where λ_1 and λ_2 are constants, and corresponds to a special case of the form considered by Matyjasek (1995). Third, on dimensional grounds again, λ can be written as $\lambda \propto \frac{1}{\ell_{Pl}}\left(\frac{\ell_{Pl}}{a}\right)^n$,

where $\ell_{Pl} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$ is the Planck lenght. In a classical theory such as General Relativity an $\hbar$ dependence in λ is not expected. The correct choice for n, therefore, is 2 (Chen &Wu 1990; Waga 1993). We have included the constant term in eq.(6) so as to be able to compare the variable term with it.

Denoting by ρ_B the sum of luminous and non luminous (dark) baryonic energy densities, and by ρ_{NB} the sum of non baryonic energy densities the total matter density is $\rho_M = \rho_B + \rho_{NB}$. The main argument for baryonic dark matter (see Carr 1994 for a review) is associated with the successful calculations in the standard model of the primordial abundances of light elements $[X(^4He) \approx 0.24, X(^2D) \sim X(^3He) \sim 10^{-5}, X(^7Li) \sim 10^{-10}]$. These predictions apply only if the baryon density parameter lies in the range $0.009h^{-2} \leq \Omega_B \leq 0.02h^{-2}$ (Copi, Schramm, & Turner 1995; Malaney & Mathews 1993, Walker *et al.* 1991). On the other hand, the density parameter $\Omega_B^{lum} \sim 0.01$ of the luminous baryons is certainly below this range. Therefore there must be a significant amount of nonluminous baryonic matter in the universe.

To proceed further, we need to make an assumption about the interaction of matter with the variable vacuum energy. There are two possibilities leading to two distinct models.

3.1. Model 1

Matter and the time-dependent vacuum do not interact with each other. In this case the energy conservation equation

$$d[\rho_M(t)a^3 + \rho_\Lambda(t)a^3] + [p_M(t) + p_\Lambda(t)]da^3 = 0 \tag{8}$$

with $p_M(t) = 0$ leads to

$$d[\rho_M(t)a^3] = 0, \tag{9a}$$

$$d[\rho_\Lambda(t)a^3] + p_\Lambda(t)da^3 = 0, \tag{9b}$$

from which we obtain $\rho_M(t) = \rho_M a_0^3/a^3$ and $p_\Lambda(t) = -C_1\rho_M - \frac{1}{3}C_2\rho_M a_0^2/a^2$ [4]. The Friedmann equation (5) then becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_M\left[\frac{a_0^3}{a^3} + C_1 + C_2\frac{a_0^2}{a^2}\right] - \frac{k}{a^2}, \tag{10a}$$

$$= \frac{8\pi G}{3}\rho_M\left[\frac{a_0^3}{a^3} + C_1\right] - \frac{k_{eff}}{a^2}, \tag{10b}$$

where we have defined

$$k_{eff} = k - \frac{8\pi G}{3}C_2\rho_M a_0^2$$
$$= k - C_2\Omega_M H_0^2 a_0^2. \tag{11}$$

Eq.(10b) suggests that the fate of the universe in regard to whether it will expand forever or collapse in the future is determined not by k, as opposed to the standard model, but by k_{eff}. This is a rather interesting feature of the models with $\lambda(t)$ varying as a^{-2}. Unfortunately, this feature has not been appreciated in the literature well enough. It seems from eq.(11) that the curvature constant may take on any of the three values 1, 0, and -1. If, however, one desires a universe that does not suffer from the initial singularity, horizon , and entropy problems, one must then consider the extension of this model to the very early universe. There one finds that with a vacuum energy decaying as a^{-2} the universe does not suffer from these problems only if $k = 1$ (Özer & Taha 1986, 1987). Note the intriguing possibility that the universe simulates the expansion dynamics of a flat universe even though it is closed ($k = 1$). This occurs for $k_{eff} = 0$, which is realized if

$$C_2 = \frac{3}{8\pi G\rho_M a_0^2} = \frac{1}{\Omega_M H_0^2 a_0^2}. \tag{12}$$

Expressing eq.(10b) in terms of the present quantities yields

$$\frac{k_{eff}}{H_0^2 a_0^2} = \Omega_M(1 + C_1) - 1. \tag{13}$$

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[4]Stipulating that in a locally inertial frame the vacuum energy-momentum tensor be Lorentz invariant requires that it be proportional to the Minkowski metric tensor $diag(-1, 1, 1, 1,)$ as this is the only 4x4 matrix that is invariant under Lorentz boosts. On the other hand, the energy-momentum tensor of a perfect fluid is of the form $diag(\rho, p, p, p)$. Hence vacuum must be a perfect fluid with the equation of state $p_{vac} = -\rho_{vac}$. In general, this does not require that the vacuum energy and hence the cosmological constant be time-independent. ρ_{vac} may be varying with the time. To qualify as a vacuum energy it suffices for ρ_{vac} to satisfy the above vacuum equation of state. However, in the literature a time-varying cosmological term that does not strictly satisfy $p_{vac} = -\rho_{vac}$ has also been called "time-varying cosmological constant" or "time-varying vacuum energy" (Peebles & Ratra 1988, Ratra & Peebles 1988, Ratra & Quillen 1992, Peebles 1993). Our Model 1 does not satisfy the vacuum equation of state if $C_2 \neq 0$. We shall, however, consider later a model that satisfies it as a special case (see Model 2, section 3.2)

On the other hand, if C_2 is greater than that in eq.(12) k_{eff} will be negative and the expansion dynamics of the universe will be similar to that of an open universe.

3.1.1. Confrontation of Model 1 with Gravitational Lensing and Supernova Studies

A time-independent cosmological constant has usually been invoked for two purposes. First, for large H_0, to increase the age of the universe to the level of the pre *Hipparcos* globular cluster age. Second, to obtain a spatially flat universe for low Ω_M, as generally implied by inflationary models, so that $\Omega_M + \Omega_\Lambda = 1$. But does a non-vanishing time-independent cosmological constant really exist? Hence it is most important to search for ways in which its existence can be tested. Fukugita, Futamase & Kasai (1990) have argued that a statistical study of gravitational lenses could provide for such a test. They have pointed out that with a cosmological constant the gravitational lensing optical depth (integrated probability) increases very rapidly as the source redshift increases. However, they do not make any direct comparison to observational data. This has been attempted by Turner (1990). Using the available data on the frequency of multiple image lensing of high-redshift quasars by galaxies Turner (1990) has shown that spatially flat $k = 0$ models with small values of Ω_M and correspondingly large values of Ω_Λ all predict much larger values of the gravitational optical depth. He then concludes that if $k = 0$ then $\Omega_M = 1$ and $\Omega_\Lambda = 0$ seems to be the favored possibility. Later on, Kochanek (1993,1995) and Maoz & Rix (1993) have managed to put upper bounds on using the statistics of lenses. Kochanek (1993, 1995) finds that in a flat universe the upper limit on Ω_Λ can be as high as 0.8 or as low as 0.65. The investigations of Maoz & Rix (1993) constrain Ω_Λ to be ≤ 0.7, also for a spatially flat universe, and lead them to conclude robustly that a (time-independent) cosmological constant no longer provides an attractive solution for the age problem of the universe then. Recently, the supernova magnitude-redshift approach has given a value for Ω_Λ somewhat smaller than the gravitational lens upper limit of Kochanek (1993, 1995). Using the initial seven of more than 28 supernovae discovered, the Supernova Cosmology Project has also measured Ω_M and Ω_Λ (Perlmutter *et al.* 1997). They find $\Omega_M = 0.88^{+0.69}_{-0.60}$ for a $\lambda = 0$ cosmology, and $\Omega_M = 0.94^{+0.34}_{-0.28}$ and $\Omega_\Lambda = 0.06^{+0.28}_{-0.34}$ for a flat universe. They find that $\Omega_\Lambda < 0.51$ at the 95% confidence level. They, too, conclude that the results for Ω_Λ-versus-Ω_M are inconsistent with λ-dominated, low density, flat cosmologies that have been proposed to reconcile the (pre *Hipparcos*) ages of globular cluster stars with large Hubble constant values. For the more general case of a Friedmann-Lemaître cosmology with the sum of Ω_M and Ω_Λ unconstrained, they find the lower limit $\Omega_\Lambda > -2.3$ and the upper limit $\Omega_\Lambda < 1.1$ for $\Omega_M \leq 1$, or $\Omega_\Lambda < 2.1$ for $\Omega_M \leq 2$. Their limits are significantly tighter than the previous limits of Carroll, Press & Turner (1992).

We next examine whether our Model 1 passes the test of gravitational lensing. The integrated probability, the so-called optical depth, for lensing by a population of singular isothermal spheres of constant comoving density normalized by the fiducial case of the SM, the Einstein-de Sitter model, is

$$P_{lens} = \frac{15}{4}\left[1 - \frac{1}{(1+z_s)^{1/2}}\right]^{-3} \int_0^{z_s} \frac{(1+z)^2}{E(z)} \left[\frac{d(0,z)d(z,z_s)}{d(0,z_s)}\right]^2 dz \tag{14}$$

(Carroll, Press, & Turner 1992) [5], where

$$E(z)^2 = (1+z)^2(1+z\Omega_M) - z(z+2)\Omega_\Lambda \tag{15}$$

and is defined by

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 E(z)^2 \tag{16}$$

(Peebles 1993). Note that $P_{lens} = 1$ for the Einstein-de-Sitter model (in which $\Omega_M = 1$, $\Omega_\Lambda = 0$) . $z = (a_0/a) - 1$ is the redshift and z_s is the redshift of the source (quasar). The angular diameter distance from redshift z_1 to redshift z_2 is given by

$$d(z_1, z_2) = \frac{1}{(1+z_2) \mid \Omega_k \mid^{1/2}} sinn\left[\mid \Omega_k \mid^{1/2} \int_{z_1}^{z_2} \frac{dz}{E(z)}\right] \tag{17}$$

where $\Omega_k = -k/(H_0^2 a_0^2)$ and "$sinn$" is defined as $sinh$ if $\Omega_k > 0$, as sin if $\Omega_k < 0$ and as unity if $\Omega_k = 0$ in which case the $\mid \Omega_k \mid^{1/2}$'s disappear from eq.(17). Equation (10a) or (10b) gives

$$E(z)^2 = (1+z)^2(1+z\Omega_M) - z(z+2)C_1\Omega_M, \tag{18}$$

where we have used the constraint

$$\begin{aligned} \Omega_M + C_1\Omega_M + C_2\Omega_M + \Omega_k &= 1 \\ \Omega_M + C_1\Omega_M + \Omega_{k_{eff}} &= 1 \end{aligned} \tag{19}$$

to eliminate Ω_k. It is thus seen upon comparing eq.(18) with eq.(15) that we only need to replace Ω_Λ with $\Omega_{C_1} = C_1\Omega_M$ to convert the expressions (14) and (17) to this model.We present in Table 1 the normalized optical depths in the Friedmann model with $k = 0$ for a typical source redshift of $z_s = 2$.

Taking the upper bound on Ω_Λ as 0.5 (Perlmutter *et al.* 1997) we see that the corresponding lensing prediction is $P_{lens} = 1.92$. Hence, we shall assume in the following

[5]It is assumed in eq.(14) that the present matter pressure is negligible; hence $\rho_M(t) = \rho_M a_0^3/a^3$. It does not hold in models with nonzero matter pressure at present.

that the maximum tolerable value is about 2. The predictions increase very rapidly for larger source redshifts. We present a sample from our predictions for $k = 1$ and $k_{eff} = 0$ in Figure 2 subject to the constraint equation (19) from which it follows in this case that $C_1 = 1/\Omega_M - 1$ and $C_2 = -\Omega_k/\Omega_M$. It is seen that plausible P_{lens} values are obtained only for $\Omega_M \geq 0.5$, or equivalently for $\Omega_{C_1} \leq 0.5$ with the corresponding C_1 values being ≤ 1 only.

3.1.2. Age of the Universe in Model 1

Inserting eq.(13) in eq.(10b) we obtain the relation between the present age t_0 of the universe and the present value of the Hubble's constant H_0:

$$H_0 t_0 = \Omega_M^{-1/2} \int_0^1 y^{1/2} \{1 + C_1 y^3 + [\Omega_M^{-1} - (1 + C_1)]y\}^{-1/2} dy. \tag{20}$$

It is worth noting that this expression is independent of C_2, namely the age of the universe in Model 1 is independent of the part of ρ_Λ varying as a^{-2} (However, see Model 2 below). For $k_{eff} = 0$ ($\Omega_M + C_1\Omega_M = 1$) eq.(20) reduces to

$$H_0 t_0(k_{eff} = 0) = \frac{2}{3(1 - \Omega_M)^{1/2}} \sinh^{-1}(\Omega_M^{-1} - 1)^{1/2}, \tag{21}$$

which is identical to that in a universe with $k = 0$. With $H_0 = 100h kms^{-1}Mpc^{-1}$ the age is $t_0 = (9.78/h)(H_0 t_0)Gyr$, where $H_0 t_0$ is calculated from equations (20) or (21), depending on the value of k_{eff} .

Next we address ourselves the question of whether the ages in the SM that are below the *Hipparcos* lower limit of $11Gyr$ in Figure 1 could be raised to $11Gyr$ with the help of a decaying cosmological term as considered in this section. Starting with $h = 0.60$, we have determined the value of C_1, by trial and error from eq.(20), that gives the age as $11Gyr$ for a certain Ω_M. Then we have calculated the corresponding lensing prediction as a function of C_2. The values thus obtained are presented in Figure 3 for $h = 0.60$ and 0.65 and in Figure 4 for $h = 0.70$ and 0.80. Disqualifying P_{lens} values that are significantly over 2, as we have decided before, it is seen from Figure 4 that $h = 0.70$ and $\Omega_M \geq 0.9$, and $h = 0.80$ and $\Omega_M > 0.2$ are unsuccessful. The time-independent component of ρ_Λ cannot help increase the age for the troublesome values of h and Ω_M.

Even though the range of Ω_M was extended up to 2 in Figure 1, we have not extended its range beyond 1 in our other calculations because the current estimates usually give a value between 0.1 to 1. For example, early dynamical estimates of the clustered mass density suggested $\Omega_M = 0.2 \pm 0.1$ (Peebles 1986; Brown & Peebles 1987). Recent studies

of galaxy clusters give $\Omega_M = 0.19 \pm 0.06$(Carlberg, Yee, & Ellingson 1997), which is comparable to the least action principle result of $\Omega_M = 0.17 \pm 0.10$ (Shaya, Peebles, & Tully 1995). On the other hand, the methods that sample the largest scales via peculiar velocities of galaxies and their production through potential fluctuations yield values of Ω_M close to unity (Dekel, Burnstein, & White 1996).

3.1.3. Dark Matter from the Decaying Vacuum Energy

Before we consider the implications of a different assumption for the interaction of matter and the vacuum next, we note that equation (19) suggests that the decaying part of ρ_Λ can be interpreted as the energy density of dark matter (nonluminous matter) with

$$\Omega_{DM} = C_2 \Omega_M. \tag{22}$$

This is similar to the dark matter from a homogeneous scalar field (Ratra & Peebles 1988; Peebles 1993). Note also that part of Ω_M may actually be due to the decaying part of ρ_Λ. Since there is no way of knowing this we have used the observational limits for Ω_M in our calculations. This will not change our conclusions in any way. If the ideas we propose here are correct there must be dark matter due to ρ_Λ not only around the galaxies but also in between the galaxies. Therefore we predict much more dark matter especially in between the galaxies than there is around the galaxies. Thus, despite the fact that the C_1 part of ρ_Λ cannot offer a satisfactory solution to the new age problem, it can help close the universe, even though by a small amount allowed by gravitational lensing, together with the C_2 part so that $k = 1$ provided $\Omega_M(1 + C_1 + C_2) > 1$, or produce a $k_{eff} = 0$ or -1 universe. We have also checked that a vanishing or a negative C_1 gives better lensing predictions. But negative values of C_1 not only necessitate higher values of C_2, hence more dark matter would be required to close the universe, but also aggravate the age problem. A purely constant term due to a relic time-independent cosmological constant, like the C_1 component of ρ_Λ, was considered previously by Peebles (1984) and Turner, Steigman & Krauss (1984) to obtain a flat $k = 0$ universe. Such a term is spatially constant and cannot be considered dark matter, even though it modifies the total energy density parameter $\Omega_T = \Omega_M + \Omega_\Lambda$. A spatially nonuniform energy density, however, can serve as dark matter.

3.2. Model 2

Matter and the time-dependent vacuum interact with each other as governed by the equation

$$d[\rho_M(t)a^3] + d[\rho_\Lambda(t)a^3] + w\rho_\Lambda(t)da^3 = 0 \tag{23}$$

where $p_M(t) = 0$ and $p_\Lambda(t) = w\rho_\Lambda(t)$. Here w is a negative parameter that is greater than or equal to - 1. The case with $w = -1$ is of great interest in that it shows that the vacuum equation of state may be realized with a decaying cosmological term. This is rendered possible due to the interaction of the vacuum with matter and is not allowed, for instance, in our Model 1 in which the vacuum and matter do not interact. Substituting eq.(6) into eq.(23) yields

$$\rho_M(t) = [1 + (w+1)C_1 + (3w+1)C_2]\rho_M\frac{a_0^3}{a^3} - (3w+1)C_2\rho_M\frac{a_0^2}{a^2} - (w+1)C_1\rho_M, \tag{24}$$

where the a^{-2} term is due to the decay of the vacuum into matter. At this point we must ask 'What particles does the vacuum decay into?'. One can only speculate. The vacuum may be decaying into various forms of matter such as baryons and axions. The Friedmann equation (5) now becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_M\left\{[1 + (w+1)C_1 + (3w+1)C_2]\frac{a_0^3}{a^3} - wC_1\right\}, \tag{25}$$

where now

$$\begin{aligned} k_{eff} &= k - 8\pi G \mid w \mid C_2\rho_M a_0^2 \\ &= k - 3 \mid w \mid C_2\Omega_M H_0^2 a_0^2, \end{aligned} \tag{26}$$

and

$$\frac{k_{eff}}{H_0^2 a_0^2} = \Omega_M[1 + C_1 + (3w+1)C_2] - 1. \tag{27}$$

Having presented the salient features of this model we next test it against gravitational lensing.

3.2.1. Gravitational Lensing and Age of the Universe in Model 2

It follows from eq.(25) that $E(z)^2$ is now given by

$$E(z)^2 = (1+z)^2(1 + z\Omega_{M_{eff}}) - z(z+2)\Omega_{C_1}, \tag{28}$$

where

$$\begin{aligned} \Omega_{M_{eff}} &= [1+(w+1)C_1+(3w+1)C_2]\Omega_M, \\ \Omega_{C_1} &= -wC_1\Omega_M. \end{aligned} \tag{29}$$

This time, equations (14) and (17) are converted to the present model under the replacement $(\Omega_M, \Omega_\Lambda, \Omega_k) \rightarrow (\Omega_{M_{eff}}, \Omega_{C_1}, \Omega_k)$. Hence, the two set of parameters having the same values will yield the same lensing predictions. In this case, a closed universe will have the same expansion dynamics of a flat one if $\Omega_{k_{eff}} = 1-[1+C_1+(3w+1)C_2]\Omega_M = 0$, which is realized if

$$C_2 = 1/(\mid w \mid 8\pi G\rho_M a_0^2) = 1/(3 \mid w \mid \Omega_M H_0^2 a_0^2). \tag{30}$$

The relation between t_0 and H_0 now is

$$\begin{aligned} H_0 t_0 = \Omega_M^{-1/2} \int_0^1 y^{1/2}\{1+(w+1)C_1+(3w+1)C_2 - wC_1y^3 \\ +[\Omega_M^{-1}-1-C_1-(3w+1)C_2]y\}^{-1/2}. \end{aligned} \tag{31}$$

An investigation of the integrand in eq.(31) shows that for the integral to be real valued $(w+1)C_1+(3w+1)C_2 > -1$.

This model has an interesting property not shared by the previous one. For $k_{eff}=0$ and $C_1=0$ it has $\Omega_{M_{eff}}=1$, and hence the same prediction for the age of the universe as the fiducial case of the SM, the Einstein-de Sitter model. This is true for all values of Ω_M satisfying $\Omega_M + (1/3 \mid w \mid -1)/(H_0^2 a_0^2) = 1$, which follows from equations (29) and (30). It is of interest to note that if $\mid w \mid < 1/3$ a universe with $\Omega_M < 1$ and $k_{eff}=0$ remains a possibility in this model with $C_1=0$. (If $\mid w \mid = 1/3$ it is necessary that $\Omega_M = 1$ so that $k_{eff}=0$.) The age of the universe in this case is then $t_0 = (6.52/h)Gyr$ (see eq.(3b)). Thus this case can survive only if the parameter h is ≤ 0.60 (see Figure 1). For $k_{eff}=0$ eq.(31) reduces to

$$\begin{aligned} H_0 t_0(k_{eff}=0) &= \frac{2}{3(\mid w \mid C_1\Omega_M)^{1/2}} \sinh^{-1}\left(\frac{\mid w \mid C_1\Omega_M}{1-\mid w \mid C_1\Omega_M}\right)^{1/2} \\ &= \frac{2}{3(1-\Omega_{M_{eff}})^{1/2}} \sinh^{-1}(\Omega_{M_{eff}}^{-1}-1)^{1/2} \end{aligned} \tag{32}$$

for positive C_1, and to

$$\begin{aligned} H_0 t_0(k_{eff}=0) &= \frac{2}{3(\mid w \mid C_1\Omega_M)^{1/2}} \sin^{-1}\left(\frac{\mid w \mid C_1\Omega_M}{1+\mid w \mid C_1\Omega_M}\right)^{1/2} \\ &= \frac{2}{3(1-\Omega_{M_{eff}})^{1/2}} \sin^{-1}(\Omega_{M_{eff}}^{-1}-1)^{1/2} \end{aligned} \tag{33}$$

for negative C_1. We see upon examining eq.(33) that the possibility of negative C_1 aggravates the age problem very severely. The special case with $C_1 = 0$ offers further possibilities. For $\Omega_{M_{eff}} < 1$ eq.(31) reduces to

$$\begin{aligned} H_0 t_0 &= \frac{1}{\{1-[1+(3w+1)C_2]\Omega_M\}} \\ &\times \left[1 - \frac{[1+(3w+1)C_2]\Omega_M}{\{1-[1+(3w+1)C_2]\Omega_M\}^{1/2}} \sinh^{-1}\{[1+(3w+1)C_2]\Omega_M]^{-1} - 1\}^{1/2}\right], \\ &= \frac{1}{(1-\Omega_{M_{eff}})^{1/2}} \left[1 - \frac{\Omega_{M_{eff}}}{(1-\Omega_{M_{eff}})^{1/2}} \sinh^{-1}(\Omega_{M_{eff}} - 1)^{1/2}\right]. \end{aligned} \tag{34}$$

As expected, this is similar to the SM expression for $k = -1$, eq.(3a), and k_{eff} is negative as long as $C_1 = 0$ and $\Omega_{M_{eff}} < 1$. In particular, for $w = -1/3$ and $\Omega_M < 1$ equations (31) and (34) reduce to eq.(3a). For $w = -1/3$ and $\Omega_{M_{eff}} > 1$ k_{eff} is positive and eq.(31) reduces to eq.(3c), which is the $k = 1$ case of the SM. Once again we emphasize that it is k_{eff} but not k that determines the expansion fate of the universe. In Table 2 we show the values of C_2 that are required to raise the age to $11Gyr$ against Ω_M and h for $C_1 = 0$ and $w = -1$. The lensing predictions now are very promising and it seems that values of h greater than 0.8 may be allowed. As can be seen from eq.(34) that the maximum value of $H_0 t_0$ when $C_1 = 0$ for negative k_{eff} is unity, in which case the age of the universe is $9.78/hGyr$. The noteworthy point here is that the $H_0 t_0$ values near one are obtained for all values of Ω_M(see Figure 5), whereas in the SM a value for $H_0 t_0$ as large as unity can only be obtained as Ω_M tends to zero for $k = -1$ (see eq.(3a).

In Tables 3, 4 and 5 we display the $H_0 t_0$ and P_{lens} predictions for $w = -1$, $-2/3$ and $-1/3$, respectively. A value of C_2 near 1 increases the age by 10 to $25-30$ percent over the SM as Ω_M increases from 0.1 to 1.0, while P_{lens} remains within the acceptable range for $w = -1$ and $-1/3$ but increases slightly over 2 for $w = -2/3$. The cases with $w = -1$ in Tables 2 and 3 are particularly interesting because they have a strictly vacuum equation of state, i.e. $p_\Lambda = -\rho_\Lambda$.

Finally we note that an independent estimate on $C_1 + C_2$ can be obtained as follows: Comparing eq.(6) with eq.(7) yields

$$\lambda = (C_1 + C_2)\frac{8\pi G}{c^2}\rho_M, \tag{35}$$

where we have retained the speed of light c. Dividing through by $3H_0^2$ we get

$$\Omega_\Lambda = (C_1 + C_2)\Omega_M, \tag{36}$$

which is consistent with $C_1 + C_2 \leq 1$.

4. DISCUSSION AND CONCLUSIONS

The ongoing recent research in cosmology and related fields reveals that it is still taken for granted by most people that the cosmological constant is a true constant which does not change with time. We have shown in our phenomenological model here that the vacuum equation of state $p_{vac} = -\rho_{vac}$ may be realized with a time-dependent cosmological term (see also Carvalho, Lima, & Waga 1992). A truly time-independent cosmological constant itself introduces a very serious problem, the so called cosmological constant problem (Weinberg 1989), to which no satisfactory solution has been found so far. In view of the cosmological constant problem and the suggested solution to it (Özer & Taha 1986, 1987) we cannot think of a mechanism to reduce the constant term in eq.(6) from its very large value in the early universe to its observational limits today. Gravitational lensing statistics limit the value of such a constant term to a level that it cannot help increase the age of the universe to the *Hipparcos* lower limit of $11Gyr$ for all h up to 0.8. If the present value of the matter density parameter is low enough neither can it help to have a $k = 0$ universe in the Friedmann models. Thus it just seems redundant . We, therefore, believe very strongly that there should be no such constant component in ρ_Λ.

It is only natural to associate a temperature with the vacuum energy. Assuming that the vacuum, which had enormous energy in the early universe, was in thermal equilibrium with the matter content of the universe at least during some period in the past, it is necessary that the temperature of the vacuum decreases with the expansion of the universe (Gasperini 1987, 1988). Thus a decaying vacuum energy or equivalently a decaying cosmological constant seems to be more plausible than a time-independent one. We have shown in this work that, a decaying cosmological constant can increase the age to 11 Gyr without running into conflicts with the gravitational lensing predictions for all $h \leq 0.80$, and even for larger values provided the vacuum and matter interact with each other (as in our Model 2). We have also argued here that a decaying cosmological constant can serve as dark matter. A very intriguing possibility is that a cosmological constant decaying as a^{-2} necessitates the definition of an effective curvature constant k_{eff}, which in turn may cause a closed universe (with $k = 1$) to have similar expansion dynamics as a flat universe if $k_{eff} = 0$ or as an open universe if $k_{eff} = -1$. In our opinion, this is an intriguing possibility of such cosmologies that deserve further study. However, these are not the only successes of such cosmologies. We should like to mention that these models do not suffer from cosmological problems such as the initial singularity, horizon, and entropy problems of the standard model and thus yield a problem free universe (Özer & Taha 1986, 1987).

The interesting and ultimate question is to explain how a time-dependent term, and in particular one decaying as a^{-2} can arise in the first place. A first attempt in this

direction has been undertaken by Peebles & Ratra (1988) and Ratra & Peebles (1988). They have considered a model in which the vacuum energy ρ_Λ depends on a scalar field that changes as the universe expands (see also Ratra & Quillen 1992). In the two string-motivated scalar-field cosmological models of Özer & Taha (1992) the universe is closed and non-singular. The scalar fields have a negative pressure of $-\frac{1}{3}\rho_\phi$ and $-\frac{2}{3}\rho_\phi$ in these models. Furthermore, the energy density ρ_ϕ of the scalar field decays like a^{-2}. Hence these scalar field models mimic the decaying cosmological constant models considered here. This endeavor is currently under further investigation. Other objects whose energy densities decrease as a^{-2} are the textures, which are topological defects. The most studied such objects are the non-Abelian cosmic strings (Vilenkin & Shellard 1993). If strings do not intercommute nor pass through each other, then their energy density scale as a^{-2} (Vilenkin 1984). Recently, a closed universe with $\Omega_M < 1$ and some form of matter (a scalar field, a cosmic string or some other stable texture) with an equation of state $p = -\rho/3$ and energy density ρ scaling as a^{-2} was considered by Kamionkowski & Toumbas (1996). Another work which is also similar to the present one is the work of Spergel & Pen (1996) who considered a flat universe dominated at present by cosmic strings. The universe in these two models locally resemble an open universe, namely $k_{eff} = -1$ even though $k = 1$ in the first and 0 in the second one. These authors argue that such cosmological models are currently viable and thus represent alternatives to the SM.

In conclusion, phenomenological cosmological models or their field theoretic partners based on a vacuum energy of the form $\rho_\Lambda(t) = C_1\rho_M + C_2\rho_M a_0^2/a^2$, preferably with $C_1 = 0$, can raise the age of the universe up to the *Hipparcos* lower limit of $11Gyr$ and serve as dark matter. When $C_1 = 0$ the maximum value of H_0t_0 is equal to one corresponding to $t_0 = (9.78/h)Gyr$, and there is no problem with the age even if h is as large as 0.85.

We have not attempted in this paper to study the growth of inhomogeneities in cosmological models of the type considered here. We hope to undertake this endeavor in future work.

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Table 1. Normalized optical depths in a $k = 0$ universe.

Ω_M	Ω_Λ	P_{lens}
0	1.0	13.25
0.1	0.9	5.98
0.2	0.8	3.94
0.3	0.7	2.93
0.4	0.6	2.33
0.5	0.5	1.92
0.6	0.4	1.63
0.7	0.3	1.42
0.8	0.2	1.25
0.9	0.1	1.11
1.0	0	1.00

Table 2. Values of C_2 to raise the age to $11Gyr$ in Model 2 [a,b]

	h				
Ω_M	0.8	0.75	0.70	0.65	0.60
0.1	0.01(1.71)				
0.2	0.26(1.75)				
0.3	0.15(1.59)	0.15(1.59)			
0.4	0.38(1.83)	0.24(1.63)			
0.5	0.40(1.87)	0.30(1.68)	0.14(1.45)		
0.6	0.42(1.92)	0.33(1.71)	0.19(1.46)		
0.7	0.43(1.96)	0.35(1.74)	0.24(1.50)	0.07(1.24)	
0.8	0.44(2.01)	0.37(1.78)	0.27(1.52)	0.13(1.27)	
0.9	0.44(2.04)	0.39(1.84)	0.30(1.57)	0.17(1.29)	
1.0	0.45(2.11)	0.40(1.88)	0.32(1.60)	0.20(1.31)	0.03(1.04)

[a] $C_1 = 0$, $w = -1$

[b] The numbers in parentheses are the corresponding P_{lens} values

Table 3. Ages and normalized optical depths for $w = -1$ in Model 2 [a]

Ω_M	Ω_k	$H_0 t_0$	P_{lens}
0.1	0.855	0.98	1.89
0.2	0.71	0.97	1.91
0.3	0.565	0.95	1.93
0.4	0.42	0.94	1.95
0.5	0.275	0.94	1.98
0.6	0.13	0.93	2.00
0.7	-0.015	0.92	2.03
0.8	-0.16	0.91	2.05
0.9	-0.305	0.90	2.08
1.0	-0.45	0.90	2.11

[a] $C_1 = 0$, $C_2 = 0.45$

Table 4. Ages and normalized optical depths for $w = -2/3$ in Model 2 [a]

Ω_M	Ω_k [b]	$H_0 t_0$ [b]	P_{lens} [b]
0.1	0.855, 0.805	0.93, 0.99	1.80, 1.91
0.2	0.710, 0.610	0.89, 0.98	1.74, 1.96
0.3	0.565, 0.415	0.86, 0.97	1.68, 2.01
0.4	0.420, 0.220	0.84, 0.97	1.63, 2.07
0.5	0.275, 0.025	0.82, 0.96	1.59, 2.12
0.6	0.130, -0.170	0.80, 0.95	1.54, 2.18
0.7	-0.015, -0.365	0.78, 0.95	1.50, 2.24
0.8	-0.160, -0.560	0.77, 0.94	1.47, 2.31
0.9	-0.305, -0.755	0.75, 0.94	1.43, 2.38
1.0	-0.450, -0.950	0.74, 0.94	1.40, 2.45

[a] $C_1 = 0$

[b] The first and second numbers are for $C_2 = 0.45$ and 0.95, respectively.

Table 5. Ages and normalized optical depths for $w = -1/3$ in Model 2 [a]

Ω_M	Ω_k [b]	$H_0 t_0$	P_{lens} [b]
0.1	0.855, 0.805	0.90	1.72, 1.73
0.2	0.710, 0.610	0.85	1.60, 1.62
0.3	0.565, 0.415	0.81	1.49, 1.52
0.4	0.420, 0.220	0.78	1.40, 1.44
0.5	0.275, 0.025	0.75	1.33, 1.37
0.6	0.130, -0.170	0.73	1.26, 1.30
0.7	-0.015, -0.365	0.71	1.20, 1.25
0.8	-0.160, -0.560	0.70	1.14, 1.20
0.9	-0.305, -0.755	0.68	1.09, 1.15
1.0	-0.450, -0.950	0.67	1.05, 1.11

[a] $C_1 = 0$

[b] The first and second numbers are for $C_2 = 0.45$ and 0.95, respectively.

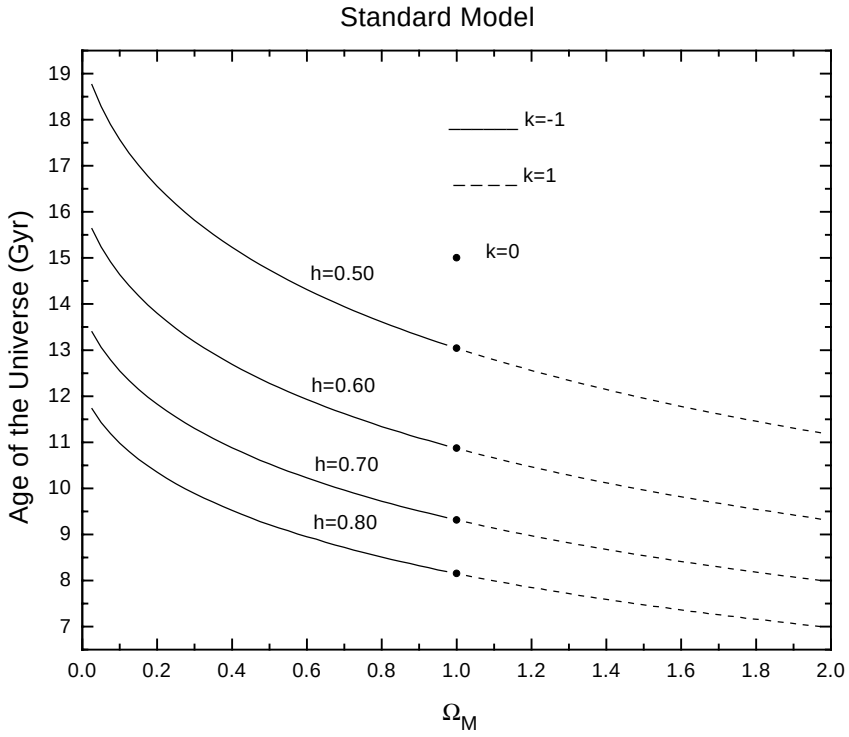


FIG.1

Fig. 1.— The age of the universe in the SM for $k = -1$ (solid lines), $k = 0$ (dots) and $k = 1$ (dashed lines) versus the present value of the matter density parameter Ω_M.

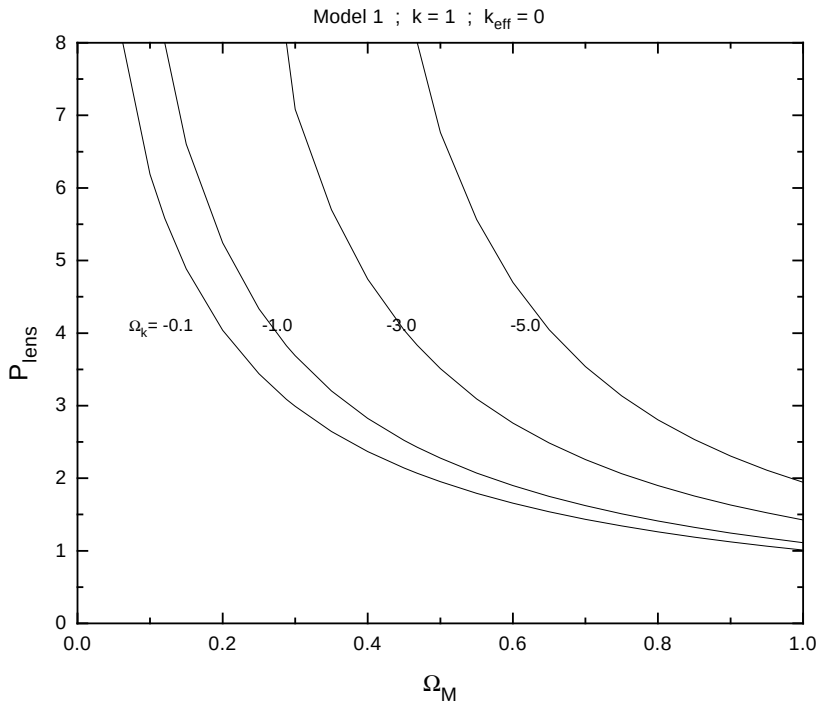


FIG.2

Fig. 2.— Plot of P_{lens} versus Ω_M in Model 1 for various values of Ω_k.

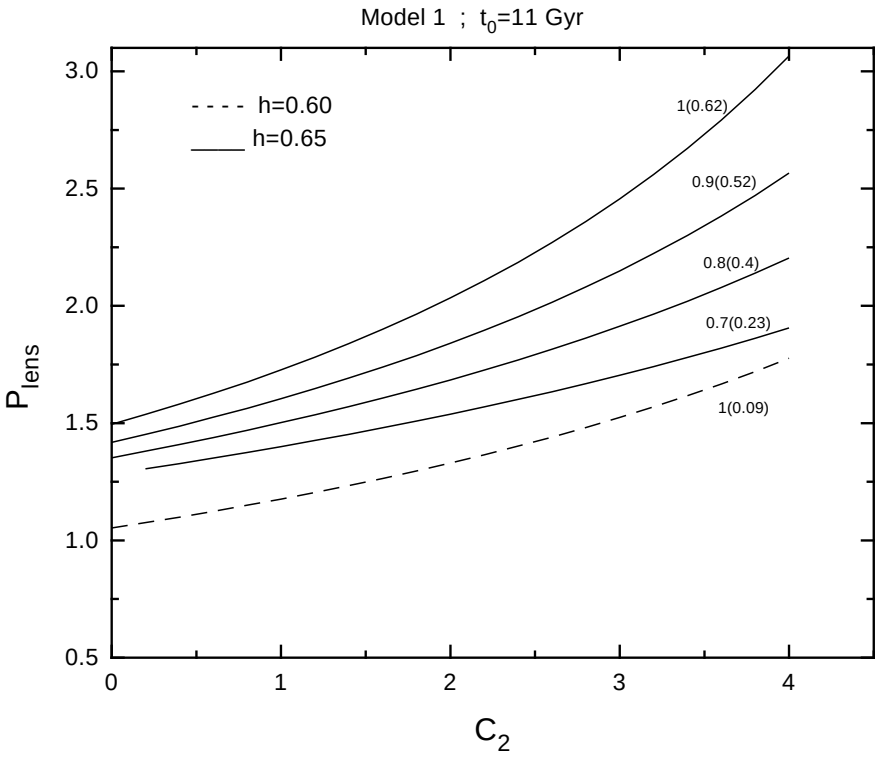


FIG.3

Fig. 3.— Plot of P_{lens} versus C_2 in Model 1 for an age of $t_0 = 11Gyr$. The first and second numbers on the curves correspond to Ω_M and C_1.

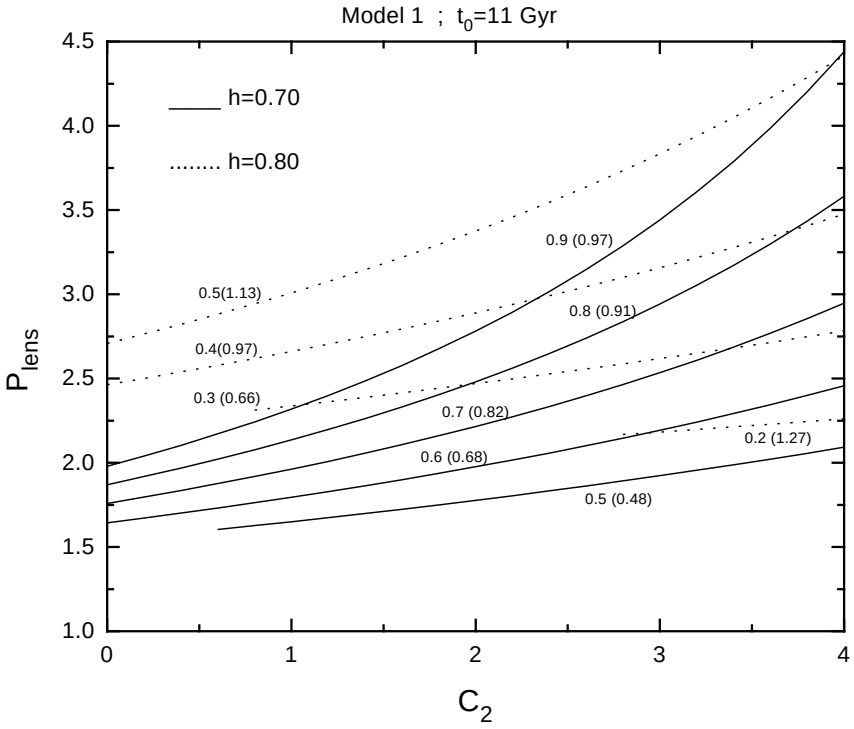


FIG.4

Fig. 4.— Plot of P_{lens} versus C_2 in Model 1 for an age of $t_0 = 11Gyr$. The first and second numbers on the curves correspond to Ω_M and C_1.

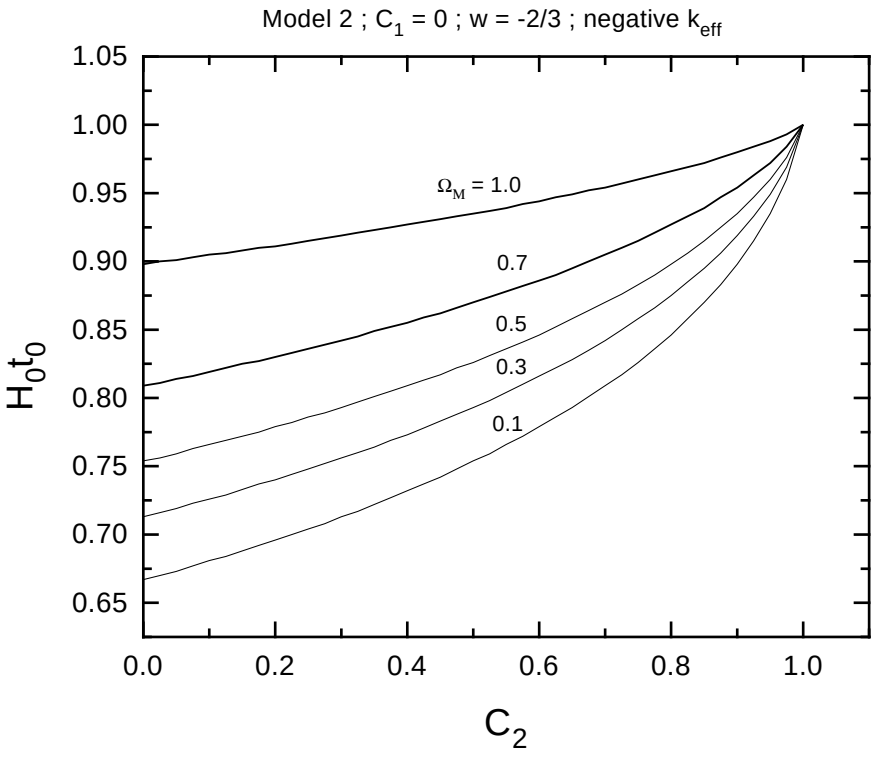


FIG.5

Fig. 5.— Plot of $H_0 t_0$ versus C_2 in Model 2 for various values of Ω_M.