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RESEARCH ARTICLE

Soliton Solutions of Some Ocean Waves Supported by Physics Informed Neural Network Method

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Abstract: In this study, we aim to obtain numerical results of the modified Benjamin-Bona-Mahony equation, Ostrovsky-Benjamin-Bona-Mahony equation and Mikhailov-Novikov-Wang equation via the physics-informed neural networks (PINN) method. The equations are modeled for shallow and long water waves, as well as fundamental and phenomenonal models in ocean engineering. According to the implementation, we obtained the PINN solutions of kink, bright, multisoliton (two-soliton) and mixed dark-bright soliton solutions. According to the inference from the obtained results, we achieved good results in some cases compared to other approximate solution methods in the literature. However, it was also observed that the best possible results could not be obtained in cases where the soliton type was intricate and layered. While the results were obtained, the number of hidden layers and the number of neural networks in the layers also varied. These results are shown in tables. Since it is known that the aforementioned models are not solved by the PINN method, we anticipate that the study will lead to other studies in the field of ocean engineering.

Keywords: soliton solutions, modified Benjamin-Bona-Mahony equation, Ostrovsky-Benjamin-Bona-Mahony equation, Mikhailov-Novikov-Wang equation, physics informed neural networks

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1. Introduction

Machine learning and artificial neural networks draw attention to the advantages they provide and the widening of their usage areas. Machine learning and artificial neural networks, which are extensively used in scientific studies, are helpful technologies for researchers to clarify time and focus. One of these fields is differential equations [1, 2].

Differential equations are used effectively in water wave models. This situation is becoming an area of interest for researchers in the integration of machine learning and artificial neural networks, which are today's technologies, into these areas. The physics-informed neural networks (PINN) model proposed by Raissi et al. to solve nonlinear PDEs can also be evaluated in this context[3]. The PINN model, which is one of the models with increasing potential in mathematical physics and engineering, was used in this study with the support of artificial neural networks in physically based solutions of ocean waves.

Some of the studies used by PINN to obtain numerical solutions are as follows: The PINN method was discussed by Fang et al. within the scope of different soliton solutions of the Schrodinger equation (NLSE) [4]. On the other hand, PINN method is used to examine the propagation of solitons in water depending on the KdV equation [5]. In a different study, Wu et al. investigated the standard nonlinear Schrodinger equation with the PINN method based on artificial neural networks they developed and conducted studies on estimating the coefficients [6]. In the study [7], investigations were made on a rigged

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periodic wave, breath wave, soliton wave, and periodic wave solutions using the Chen – Lee – Liu equation. In [8], Sawada-Kotera equations were analyzed with the PINN method. Also, there are some different versions of the PINN method in the literature, such as multiple parallel subnets PINN (MPS-PINN) [9], improved PINN (IPINN) [10], coupled automatic-numeric PINN (CAN-PINN) [11] and energy conservation deep learning method [12] etc. On the other hand, researchers can also find some recent studies about obtaining soliton solutions to wave equations, such as [13, 14]. The method is a machine learning technique, and since it is quite new, it also raises some questions. One of these is the stability and convergence analysis. Unfortunately, the subject has not been examined in this study, but readers can take a look at some studies on this subject [15 – 18].

In this study, we investigated some shallow and long wave models, such as the modified Benjamin-Bona-Mahony (MBBM) equation, Ostrovsky-Benjamin-Bona-Mahony (OS-BBM) equation, and the Mikhailov-Novikov-Wang (MNW) equation. Some studies about MBBM can be listed as follows: the traveling wave solution of the model via the modified simple equation method [19], soliton solutions via the bifurcation method [20], analytic treatment of the model [21]. Also, researchers can find studies about OS-BBM in the literature, such as dark and bright soliton solutions of model [22]. In addition, studies like multi-soliton solutions of MNW [23] and novel exact solutions of MNW [23] can be listed as recent studies about MNW in the literature.

The organization of the paper is as follows: section 2 includes some preliminaries, as in section 2.1, the fundamentals of the PINN method are included, and in section 2.2, the PINN scheme for the models is included. Section 3, includes implementation of the PINN method on the models. In section 4, the obtained results of the application are included via tables and figures. Lastly, section 5 includes a conclusion about the study.

2. Preliminaries

In this section, we gave some basic preliminaries about artificial neural networks and the PINN method, and we also gave Python code blocks for the desired equations.

2.1 Physics informed neural networks

According to the method, the studied equations can be generalized as follows:

$$\phi t(x,t) + N[\phi(x,t)] = 0,\ x \in [x0, x1],\ t \in [t0, t1], \tag{1}$$

where $\phi(x,t)$ is the hidden solution of the PDE and N[.] is the nonlinear operator for $\phi(x,t)$. On the other hand, the net function f can be defined by the left-hand side of eq. (1) as follows:

$$f := \phi t(x,t) + N[\phi(x,t)]. \tag{2}$$

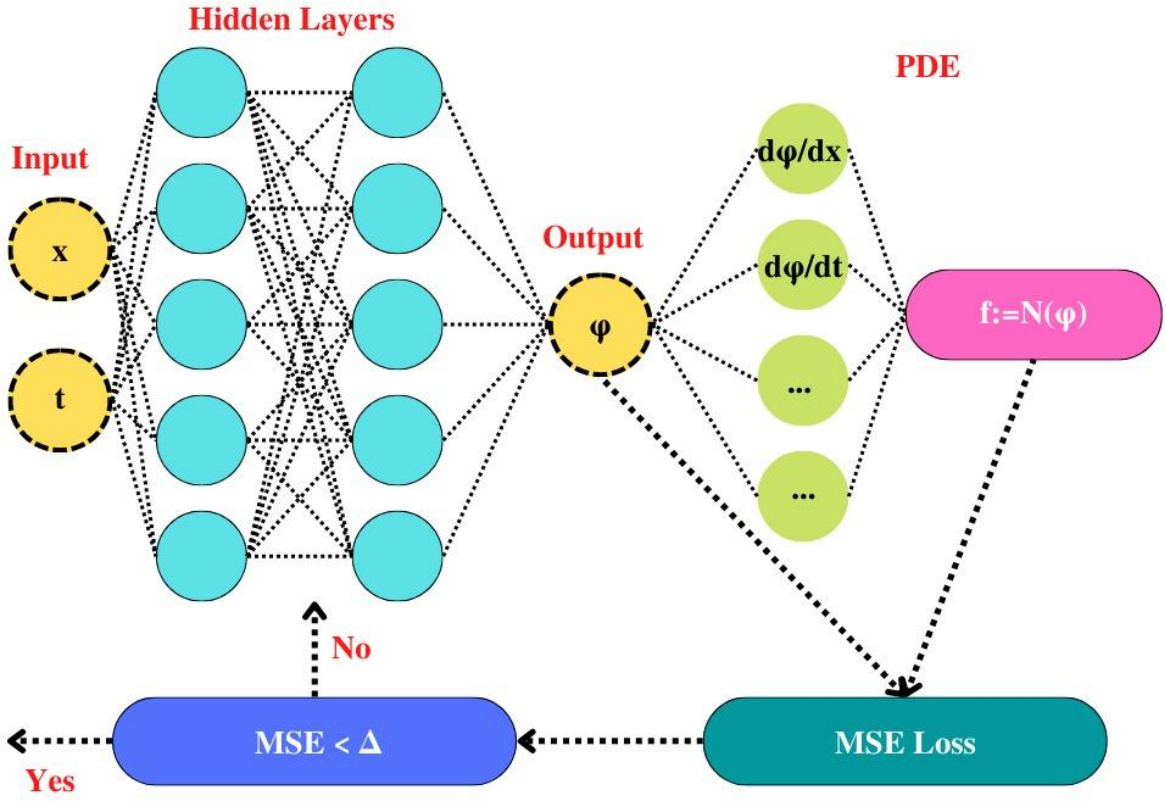
The approximation of $\phi(x,t)$ from f via minimizing mean square error (MSE) and neural networks is called PINN Minimizing MSE loss can be defined as follows:

$$\begin{aligned} MSE &= MSE_{\phi} + MSE_{f}, \\ MSE_{\phi} &= \frac{1}{N_{\phi}} \sum_{i=1}^{N\phi} \left|\phi(x_{\phi}^{i}, t_{\phi}^{i}) - \phi^{i}\right|^{2}, \\ MSE_{f} &= \frac{1}{N_{f}} \sum_{i=1}^{N_{f}} \left|f(x_{f}^{i}, t_{f}^{i})\right|^{2}, \end{aligned} \tag{3}$$

Where $\left\{x_{\phi}^{i}, t_{\phi}^{i}, \phi^{i}\right\}_{i=1}^{N_{\phi}}$ represents initial and boundary data training on $\phi(x,t)$ and $\left\{x_{f}^{i}, t_{f}^{i}\right\}_{i=1}^{N_{f}}$ represents collocation points for $f(x,t)$. The $MSE\varphi$ loss denotes the loss of initial and boundary values; the MSE f loss denotes the loss of the

created structure in eq. (2). In fig. 1, we illustrated the working scheme of the PINN method for solving PDEs. The working principle is as follows: the system is initialized using only the initial values of the PDE. Estimates obtained via initial values are substituted in f in eq. (2) and weight determinations are made by minimizing the error. The system proceeds using the previous prediction value for all points of the grid system.

Figure 1
PINN method working scheme



Also, it should not be forgotten that PINN is a machine-learning technique. In the literature, the term black box is used for systems containing artificial neural networks. However, to give a summary for readers to understand more easily, the method predicts the next step in the grid using the initial values and tries to bring the PDE solution closer to zero by substituting the prediction into the f function. This minimization process is done through the loss function. Once all steps in the grid are predicted, the surface solution of a PDE is revealed.

2.2 PINN scheme for MBBM

MBBM equation is given as follows [19];

$$\varphi t + \varphi x - \alpha\varphi 2\ \varphi x + \varphi xxx = 0, \qquad (4)$$

where α is a nonzero constant. The MBBM equation is a model for the surface of long waves in ocean engineering and represents the character of hydromagnetic waves in plasma matter [20, 21]. The real-valued neural network for the PINN scheme can be written as:

Listing 1
Real valued neural network for MBBM equation

```
def pinn_phi(self, x, t):
    phi = self.neural_net(tf.concat([x, t],1), self.weights, self.biases)
    return phi
```

Listing 2
Real valued net function for MBBM equation

```
def net_f(self, x, t):
    phi = self.pinn_phi(x, t)
    phi_t = tf.gradients(phi, t)[0]
    phi_x = tf.gradients(phi, x)[0]
    phi_xx = tf.gradients(phi_x, x)[0]
    phi_xxx = tf.gradients(phi_xx, x)[0]
    f = phi_t+phi_x-alpha*(phi**2)*phi_x + phi_xxx #Equation 1
    return f
```

2.3 PINN scheme for OS-BBM

The OS-BBM equation is given as follows[22]:

$$(\phi t + \phi x - \alpha(\phi 2)x - \beta\phi xxt)x - \gamma(\phi + \phi 2) = 0. \quad (5)$$

where α,β,γ is a nonzero constant. OS-BBM equation models wave motion on the ocean surface and some special cases of magneto-acoustic waves [22]. The real-valued neural network for the PINN scheme can be written as:

Listing 3
Real valued net function for OS-BBM equation

```
def net_f(self, x, t):
    phi = self.pinn_phi(x, t)
    phi_t = tf.gradients(phi, t)[0]
    phi_x = tf.gradients(phi, x)[0]
    phi_xx = tf.gradients(phi_x, x)[0]
    phi_xxt = tf.gradients(phi_xx, t)[0] phi_xxx = tf.gradients(phi_xx, x)[0]
    f = tf.gradients(phi_t + phi_x -alpha* tf.gradients(phi**2, x)[0] - beta* phi_xxt, x)[0] - gamma(phi + phi**2) #Equation 2
```

```
        return f
```

2.4 PINN scheme for MNW

The MNW equation is given as follows[23, 24]:

$$\phi tt - \phi xxxt - 8\phi x\phi xt - 4\phi xx\phi t + 2\phi x\phi xxxx + 4\phi xx\phi xxx + 24\phi\phi xx = 0. \quad (6)$$

MNW equations model some special ocean waves and can also be derived from other phenomenon ocean wave models [23–25]. The real-valued neural network for the PINN scheme can be written as:

Listing 4
Real valued net function for MNW equation

```
def net_f(self, x, t):
        phi = self.pinn_phi(x, t)
        phi_t = tf.gradients(phi, t)[0]
        phi_x = tf.gradients(phi, x)[0]
        phi_xt = tf.gradients(phi_x, t)[0]
        phi_xx = tf.gradients(phi_x, x)[0]
        phi_tt = tf.gradients(phi_t, t)[0]
        phi_xxx = tf.gradients(phi_xx, x)[0]
        phi_xxxt= tf.gradients(phi_xxx, t)[0]
        phi_xxxx = tf.gradients(phi_xxx, x)[0]
        f = phi_tt - phi_xxxt - 8*phi_x*phi_xt -4*phi_xx*phi_t
    + 2*phi_x*phi_xxxx + 4*u_xx*u_xxx + 24*((phi_x)**2)*phi_xx
    return f
```

3. Application

In this section, we applied the PINN method to the (1+1)-dimensional PDEs.

3.1 Modified Benjamin-Bona-Mahony equation

In study [19], the kink soliton solution of the eq. (4) given as:

$$\phi(x,t) = \sqrt{\left(\frac{3(1-\beta)}{\alpha}\right)} \times \tanh\left(\sqrt{\left(\frac{1-\beta}{2}\right)}(x-\beta t)\right), \quad (7)$$

where α,β are arbitrary real constants with $\frac{1-\beta}{2} > 0$ and also β is wave velocity. To simulate our method, we created a~~dataset of the exact solution of the eq. (7) via MATLAB. Datasets are created as follows; x and t aregenerated for 256 equally spaced values in [−10, 10] .

3.2 Ostrovsky-Benjamin-Bona-Mahony equation

In study [22], bright soliton solution of the eq. (5) given as:

$$\phi(x,t) = \frac{-3}{2}\operatorname{sech}^2\left(\frac{\sqrt{\alpha+\beta\gamma}}{2\sqrt{\beta+\alpha\beta}}(x-\left(\frac{\alpha+\alpha^2}{\alpha+\beta\gamma}\right)t)\right), \tag{8}$$

where α , β , γ are real constants, $\left(\frac{\alpha+\alpha^2}{\alpha+\beta\gamma}\right)$ is wave velocity with α+βγ > 0 and β+αβ > 0. To simulate our method, we created a dataset as follows; x is generated for 256 equally spaced values in [−10, 10] and t is generated for 256 equally spaced values in [0, 5] .

3.3 Mikhailov-Novikov-Wang equation

In study [24], kink type one-soliton solution of the eq. (6) given as;

$$\phi(x,t) = \frac{2\alpha\, e^{\alpha^3 t+\alpha x}}{1+e^{\alpha^3 t+\alpha x}}, \tag{9}$$

where α is the wavenumber and real constant, and wave speed is α^3 . The dataset is created via MATLAB with x and t generated for 256 equally spaced values in [−10, 10] for α = 1.2.

In study [24], kink type two-soliton interaction solution of the eq. (6) given as:

$$\phi(x,t) = \frac{2\alpha_1 e^{\alpha_1^3 t+\alpha_1 x} + 2\alpha_2 e^{\alpha_2^3 t+\alpha_2 x} + \frac{2(\alpha_1-\alpha_2)^2 e^{(\alpha_1+\alpha_2)x+\left(\alpha_1^3+\alpha_2^3\right)t}}{\alpha_1+\alpha_2}}{1+e^{\alpha_1^3 t+\alpha_1 x} + e^{\alpha_2^3 t+\alpha_2 x} + \frac{(\alpha_1-\alpha_2)^2 e^{(\alpha_1+\alpha_2)x+\left(\alpha_1^3+\alpha_2^3\right)t}}{\alpha_1+\alpha_2}}, \tag{10}$$

where α_1 ,α_2 are arbitrary real constants. In this problem, we applied the method to two different parametrizations. Firstly, a dataset is created for $\alpha_1 = -1.41$ and $\alpha_2 = 1.4$ values, and nodes are divided into 256 equally spaced values with x ∈ [−20, 20] and t ∈ [−5, 5]. Secondly, a dataset is created for $\alpha_1 = -1.5$ and $\alpha_2 = 0.5$ values, and nodes are divided into 256 equally spaced values with x ∈ [−20, 20] and t ∈ [−10, 10] .

In study[23], mixed dark-bright soliton solution of the eq. (6) given as:

$$\phi(x,t) = -\frac{x}{3} + \frac{t}{3} - \frac{2}{1+\cosh(-x+t)-\sinh(-x+t)}. \tag{11}$$

The dataset is created for 256 equally spaced values with x,t ∈ [−5, 5] . The next chapter describes the results using tables and figures.

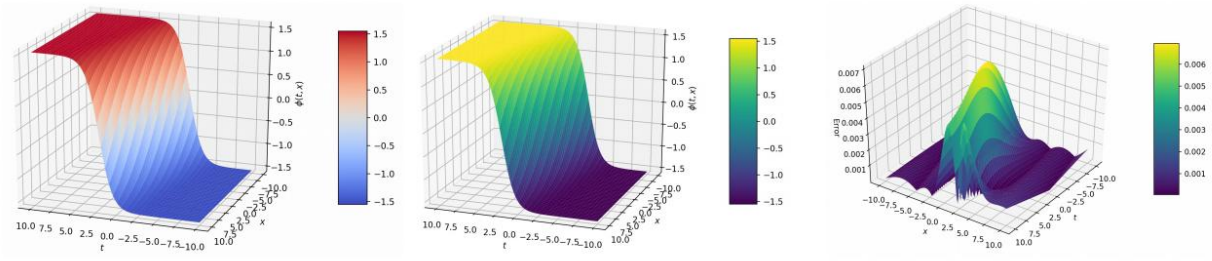
4. Results and Discussion

In this section, we give the obtained results with tables and figures. In table 1, a comparison of the obtained results of eq. (7) is given. Results are changing according to initial Nφ and neural network numbers. According to table 1, the best results are obtained with 8 hidden layers and 60 neural networks per layer. Absolute error is approximately 1, 25 × 10^{-3}. In addition, the results are shown in fig. 2. Figure 2a shows a 3D view of the exact solution; fig. 2b shows a 3D view of the approximate solution obtained via the PINN method; fig. 2c shows absolute error; and fig. 2d shows a contour plot of the approximate solution.

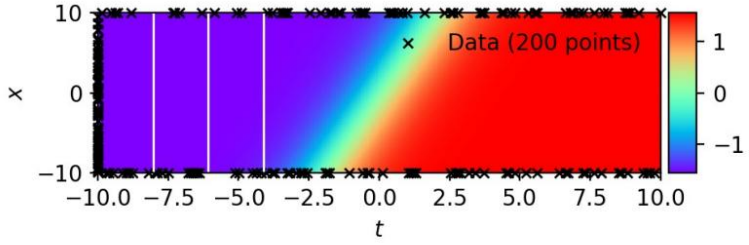
Table 1
Comparison of eq. (7)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$1,729082 \times 10^{-3}$	8,89	140
200	4000	8	40	$1,255546 \times 10^{-3}$	39,34	197
200	4000	8	60	$4,563819 \times 10^{-3}$	76,92	179
300	4000	8	100	$5,315208 \times 10^{-3}$	152,20	200

Figure 2
The various plots of φ(x,t) in eq. (7)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



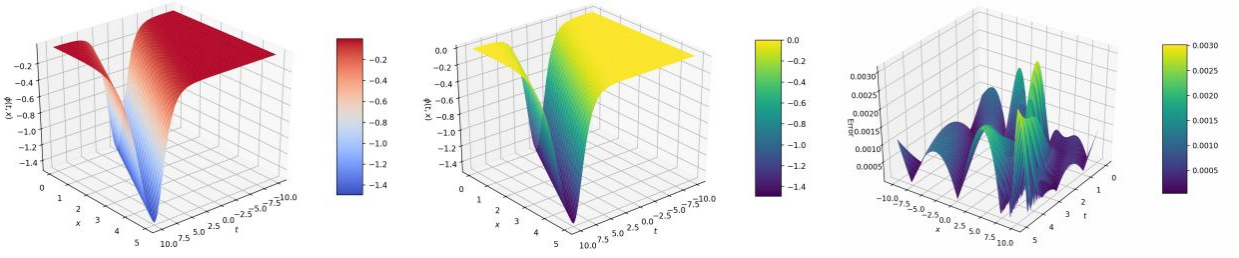
(d) Contour Plot of PINN Solution

Table 2 shows the comparison of obtained results for eq. (8). According to table 2, the best results are obtained via 8 hidden layers with 100 neural networks per layer. Absolute error is approximately 1, 82 × 10^{-3}. Figure 3 shows the results for eq. (8). In fig. 3a, the soliton solution of eq. (8) is shown. Figure 3b shows the PINN solution,and fig. 3c shows the absolute error between soliton and the PINN solution. In fig. 3d, a contour plot of the approximate solution obtained via the PINN method is depicted.

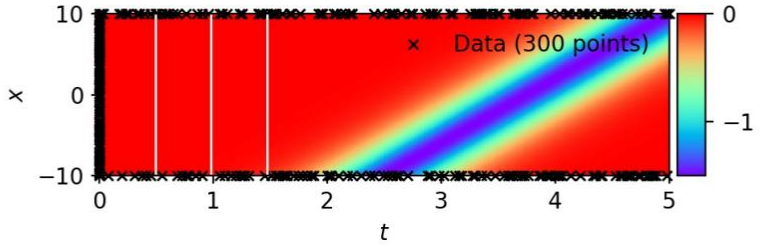
Table 2
Comparison of eq. (8)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$8,02994 \times 10^{-1}$	55,92	105
200	4000	8	40	$4,284380 \times 10^{-1}$	1499,21	371
200	4000	8	60	$7,817779 \times 10^{-1}$	776,05	151
300	4000	8	100	$1,825055 \times 10^{-3}$	1197,72	278

Figure 3
The various plots of φ(x,t) in eq. (8)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



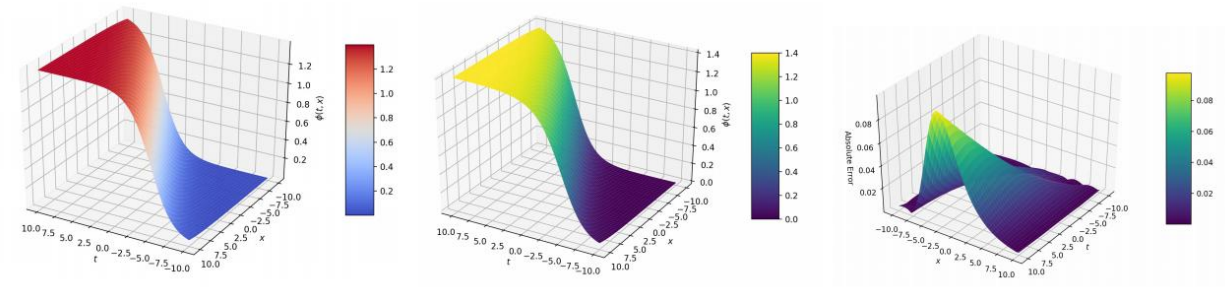
(d) Contour Plot of PINN Solution

According to table 3, the best results are obtained with 8 hidden layers and 40 neural networks per layer. The absolute error is approximately 2, 75 $\times$ 10^{-2}. In addition, the results are shown in fig. 4. Figure 4a shows a 3D view of the Soliton solution; fig. 4b shows a 3D view of the approximate solution obtained via the PINN method; fig. 4c shows the absolute error; and fig. 4d shows the contour plot of the PINN solution.

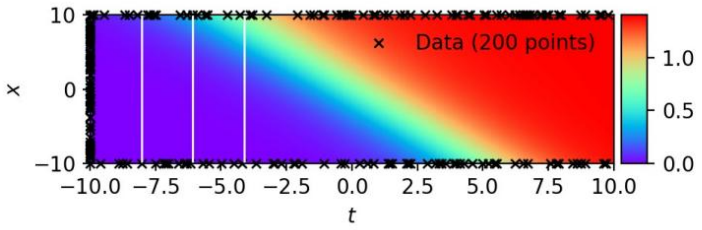
Table 3
Comparison of eq. (9)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$3,797476 \times 10^{-2}$	47,65	133
200	4000	8	40	$2,750077 \times 10^{-2}$	91,62	78
200	4000	8	60	$5,433307 \times 10^{-2}$	165,48	70
300	4000	8	100	$6,953371 \times 10^{-2}$	293,65	65

Figure 4
The various plots of φ(x,t) in eq. (9)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



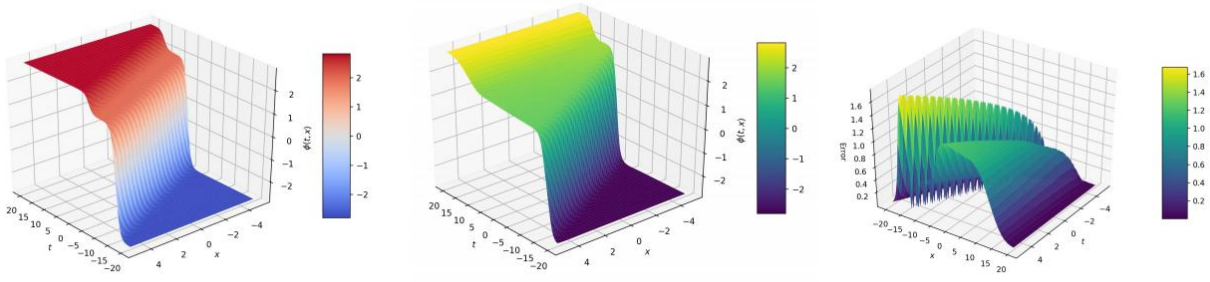
(d) Contour Plot of PINN Solution

Table 4 shows the comparison of obtained results for eq. (10). According to table 4, the best results are obtained via 8 hidden layers with 20 neural networks per layer. Absolute error is approximately $2,20 \times 10^{-2}$. Figure 5 shows the results for eq. (10). In fig. 5a, the soliton solution of eq. (10) is shown. Figure 5b shows the PINN solution, and fig. 5c shows the absolute error between the soliton solution and the PINN solution. In fig. 5d, acontour plot of the approximate solution obtained via the PINN method is depicted.

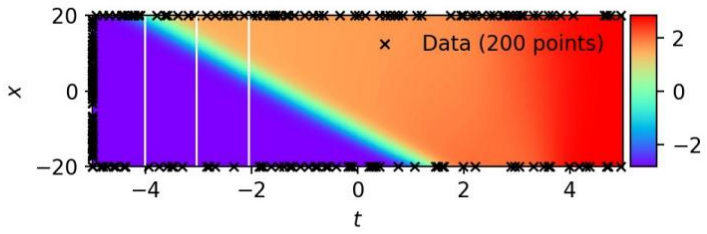
Table 4
Comparison of eq. (10)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$2,200254 \times 10^{-2}$	1536,63	6327
200	4000	8	40	$2,697991 \times 10^{-1}$	5683,23	5592
200	4000	8	60	$2,405252 \times 10^{-1}$	67103,70	7912
300	4000	8	100	$2,766252 \times 10^{-1}$	180084,39	4374

Figure 5
The various plots of φ(x,t) in eq. (10)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



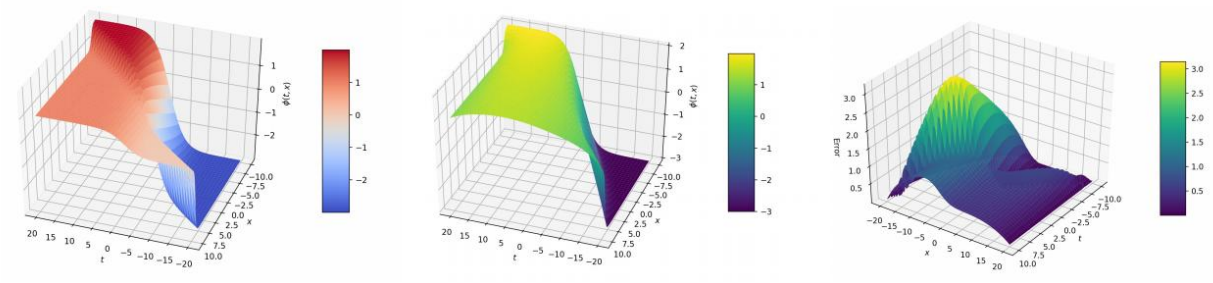
(d) Contour Plot of PINN Solution

According to table 5, the best results are obtained with 8 hidden layers and 60 neural networks per layer. The absolute error is approximately 4, 94 $\times$ 10^{-1}. In addition, the results are shown in fig. 6. Figure 6a shows a 3D view of the soliton solution; fig. 6b shows a 3D view of the approximate solution obtained via the PINN method; fig. 6c shows the absolute error; and fig. 6d shows the contour plot of the PINN solution.

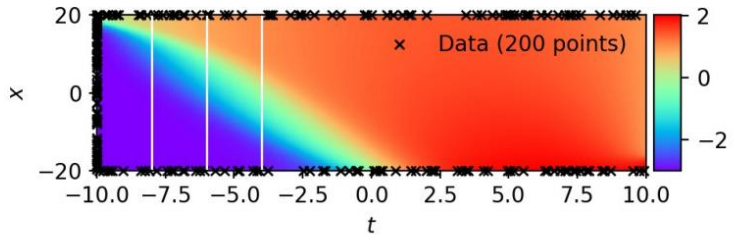
Table 5
Comparison of eq. (10)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$4,969024 \times 10^{-1}$	944,8	3796
200	4000	8	40	$5,042329 \times 10^{-1}$	7480,73	3485
200	4000	8	60	$4,944566 \times 10^{-1}$	8487,50	2805
300	4000	8	100	$5,174641 \times 10^{-1}$	78819,09	2970

Figure 6
The various plots of φ(x,t) in eq. (10)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



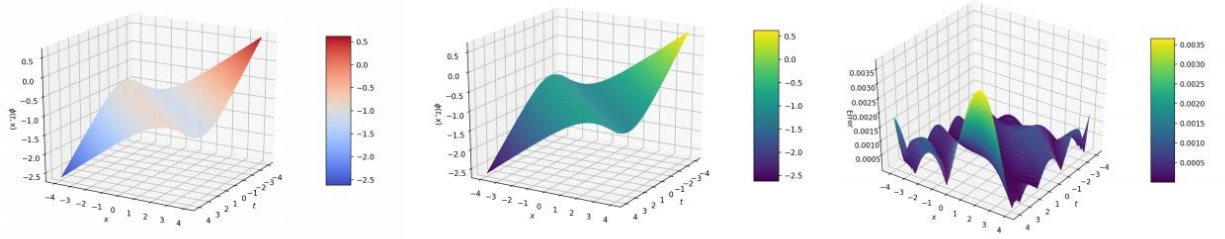
(d) Contour Plot of PINN Solution

Table 6 shows the comparison of obtained results for eq. (11). According to table 6, the best results are obtained via 8 hidden layers with 20 neural networks per layer. The absolute error is approximately $7,05 \times 10^{-4}$. Figure 7 shows the results for eq. (11). In fig. 7a, the soliton solution of eq. (11) is shown. Figure 7b shows the PINN solution,and fig. 7c shows the absolute error between the soliton solution and the PINN solution. In fig. 7d, a contour plot of the approximate solution obtained via the PINN method is depicted.

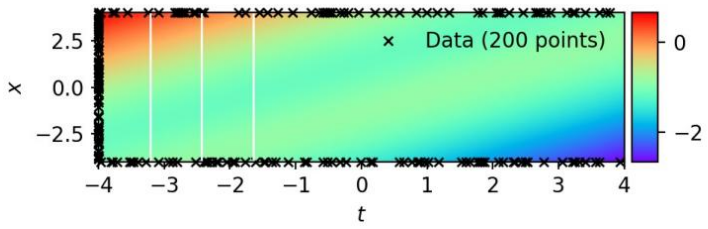
Table 6
Comparison of eq. (11)

N_ϕ	Nf	Hidden Layer	Neural Networks	Error u	Time(sec)	Number of Iterations
200	4000	8	20	$7,055715 \times 10^{-4}$	89,6	300
200	4000	8	40	$1,80200 \times 10^{-3}$	2697,15	703
200	4000	8	60	$6,029168 \times 10^{-3}$	1640,96	545
300	4000	8	100	$4,308377 \times 10^{-3}$	1307,51	369

Figure 7
The various plots of φ(x,t) in eq. (11)



(a) Soliton Solution (b) PINN Solution (c) Absolute Error



(d) Contour Plot of PINN Solution

When evaluating the results obtained in this study, we can start with the scope. When looking at the studies on obtaining solitons using PINN [4,26,27], it can be seen that the approximate error rate in existing studies is achieved. While other studies have reached an error rate of approximately 10^{-3}, these values are also present in our study. Another thing we need to compare is the types of solitons in existing studies. Looking at the study examining the two-soliton, rogue structure [26,28,29] which are the rare types in the literature, it will be seen that the error obtained is similar to the error in this study. It can be seen that the classical PINN method gives poor results compared to advanced numerical methods such as B-Spline and the modified Laplace decomposition method in layered and complex soliton types [30, 31].

Another unpredictable weakness of the PINN method is that it does not have a fixed network structure. Examining some other PINN studies revealed different error rates for varying numbers of hidden layers. Some of these studies and their results are shown in Table 7 for easier viewing.

Table 7
Comparison of results

Author	Equation	Setup	Error	Time(sec)
Fang et al. [4]	High order nonlinear Schrödinger (NLSE)	7 hidden layers × 30 neurons	9.1252×10^{-3}	5545.7
Cui et al. [32]	Boussinesq	4 hidden layers × 40 neurons	2.0×10^{-2}	152
Wang et al. [33]	Generalized Hirota-Satsuma coupled KdV	5 hidden layers × 20 neurons	6.949×10^{-5}	607.9
Peng et al. [34]	Nonlocal Hirota	9 hidden layers × 40 neurons	2.695×10^{-4}	1262.4
Yuan et al. [35]	Volterra	4 hidden layers × 80 neurons	9.17×10^{-5}	N/A

As can be seen in the Table 7, there is no correlation between the increase or decrease in the number of hidden layers or neurons and the success of the method. Therefore, researchers will have to try many times to find the most accurate result. A disadvantage can also be seen in this aspect. Another conclusion that can be drawn from the table is that the PINN method can be applied to equation types in many different fields. Although only 1D water wave models were considered in this study, other ocean engineering examples where the PINN method is used can be listed as follows: Examining the shallow water equation and advection-diffusion equation with PINN and their effects on climate change [36] and ocean temperature [37]. Examination of local advection-diffusion equations with the PINN method and uncertainty in ocean modeling [38]. Examination of internal soliton movements modeled by the Kadomtsev-Petviashvili equation with PINN and comparison of the results with satellite images [39]. In addition, when looking at the time-dependent calculation cost in Table 7, a definitive conclusion cannot be made, but there are methods in the literature that reduce the time cost, such as CAN-PINN [11]. On the other hand, another point that concerns the success of the method is the initial and boundary conditions. Some customized PINN models [40–43] have included these conditions in the loss function and achieved more successful results.

Since the PINN method is a machine learning method, we can mention some hyperparameters that affect its success. These can be listed as learning rate, activation function, optimization algorithm, number of hidden layers, and neurons.This study solely utilized tables to illustrate the impact of altering the number of hidden layers and neurons on success. However, there are currently studies showing the effect of other hyperparameters on the success of PINN. Some of these studies are as follows: the effect of hyperparameters on error convergence [44], the effect of parameters such as activation function and network architecture on the error rate [45], and some other similar studies [46–48].

5. Conclusion

In this work, we have studied approximate solutions of one-dimensional shallow water waves such as the MBBM, OSBBM,and MNW equations. Based on this work, we have identified several soliton structures of water waves, including kink, dark, kink with two solitons interacting, and mixed bright-dark solitons. On the other hand, this is the first study about the mentioned models using the PINN method. Moreover, we have reached the absolute error in the 10^{-1} to 10^{-4} interval. We understand that some of the error values obtained in our study are relatively high compared to other studies in the literature. However, some of the error values we obtained in our study also yielded reasonably good and effective results. Examining soliton types with high error rates leads to the conclusion that the complex and layered soliton structures prevent numerical solutions from yielding satisfactory results. Future researchers can set the goal of achieving better solutions in layered solitons. The solution of models used in fields such as water waves and ocean engineering, which include one-dimensional equations, as well as in the field of optics, which includes larger-dimensional and complex equations, may be one of the future goals. Another future target can be set as working on higher-dimensional or complex-valued models that are used in the optic field. Some studies have stated that they have run the system successfully and obtained results, especially for high-dimensional equations in the field of optics. However, these studies are quite incomplete compared to traditional methods in terms of absolute error. Therefore, the issue of reducing the error rate is one of the important expectations for future studies.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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