Lie group automorphisms and their connection to induced vector fields in coset manifold spaces

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Abstract

The equivalence between a control-affine system on a manifold and a linear control system on a Lie group or homogeneous space, which is obtained by a diffeomorphism, is shown to hold precisely when the vector fields of related system are complete and generate a finite-dimensional Lie algebra. The influence of Lie group automorphisms and Lie algebra derivations on the vector fields in the system is investigated, with specific focus on the necessary and sufficient conditions for extending these vector fields to be well-defined on the associated coset manifold space. The three-dimensional Heisenberg Lie group is considered, on which linear and invariant vector fields are defined. A topologically closed subgroup of this group is then examined, from which an explicit form of concerning coset manifold structure is derived. By employing the notions of automorphism and derivation on this construction, the conditions under which vector fields defined on the group can be extended in a well-defined manner to the coset space are determined.

Keywords: Linear vector fields, automorphism groups, Lie algebra derivations.
2020 Mathematics Subject Classification: 93B05, 93C05, 22E25

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