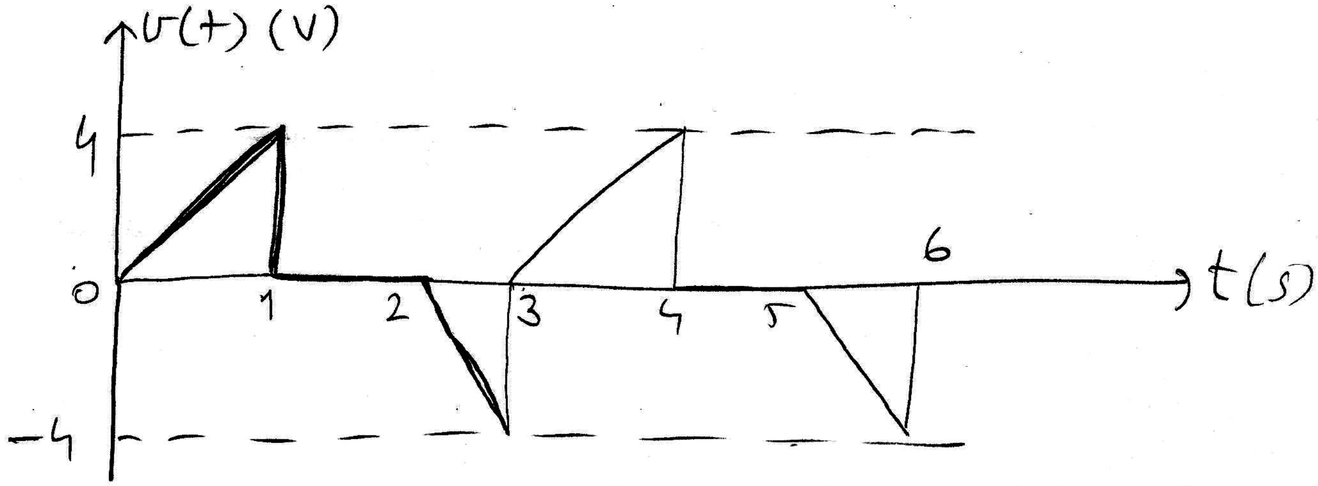
Ortalama Değer Efektif Değer Örnekler

(1)



a) $V_{ort} = ?$ b) $V_{rms} = ?$ (Efektif değer) c) Bu gerilim $R = 2\Omega$'luk dirence uygulansa $P = ?$

Çözüm

a) $T = 3$ saniye

$[0,1]$ aralığında $t=0$ için $V=0$, $t=1$ için $V=4$ $\Big\}$ $V = at+b$

$0 = 0 + b \Rightarrow b = 0$

$4 = a.1 \Rightarrow a = 4 \Rightarrow \boxed{V = 4t}$

$[1,2)$ aralığında $\boxed{V=0}$

$[2,3)$ aralığında $t=2$ için $V=0$, $t=3$ için $V=-4$ $\Big\}$ $V = at+b$

$0 = 2a + b \Rightarrow b = -2a$

$-4 = 3a + b$

$-4 = 3a - 2a \Rightarrow a = -4$, $b = 8$

$\boxed{V = -4t+8}$

fonksiyon

$$v(t) = \begin{cases} 4t & 0 < t \leq 1\text{ s} \\ 0 & 1 < t \leq 2\text{ s} \\ -4t+8 & 2 < t \leq 3\text{ s} \end{cases}$$

$$V_{ort} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{3}\left[\int_0^1 4t\,dt + \cancel{\int_1^2 0.dt}^{\,0} + \int_2^3 (-4t+8)dt\right]$$

$$V_{ort} = \frac{1}{3}\left[4.\frac{t^2}{2}\Big|_0^1 + 0 + \left(-4.\frac{t^2}{2} + 8t\right)\Big|_2^3\right]$$

$$V_{ort} = \frac{1}{3}\left[2t^2\Big|_0^1 + \left(-2t^2 + 8t\right)\Big|_2^3\right] = \frac{1}{3}\left(2.1^2 + (-2).(3^2-2^2) + 8(3-2)\right)$$

$\boxed{V_{ort}=0}$

b) $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

$V_{rms} = \sqrt{\frac{1}{3}\cdot\left[\int_0^1 (4t)^2 dt + \int_2^3 (-4t+8)^2 dt\right]}$

$V_{rms} = \sqrt{\frac{1}{3}\left[\int_0^1 16t^2 dt + \int_2^3 (16t^2-64t+64)\,dt\right]}$

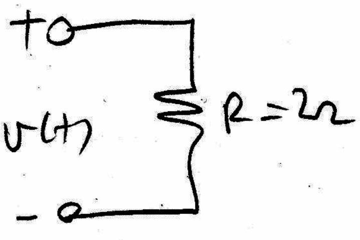
$V_{rms} = \sqrt{\frac{1}{3}\left[\frac{16}{3}t^3\Big|_0^1 + \left(\frac{16}{3}t^3-32t^2+64t\right)\Big|_2^3\right]}$

$V_{rms} = \sqrt{\frac{1}{3}\left(\frac{16}{3}+\frac{16}{3}(3^3-2^3)-32(3^2-2^2)+64(3-2)\right]}$

$V_{rms} = \sqrt{\frac{1}{3}\left[\frac{16}{3}+\frac{16}{3}\cdot 19-32\cdot 5+64\right]}$

$V_{rms} = \sqrt{\frac{1}{3}\cdot 10,667} \Rightarrow \boxed{V_{rms}=1,885\,V}$

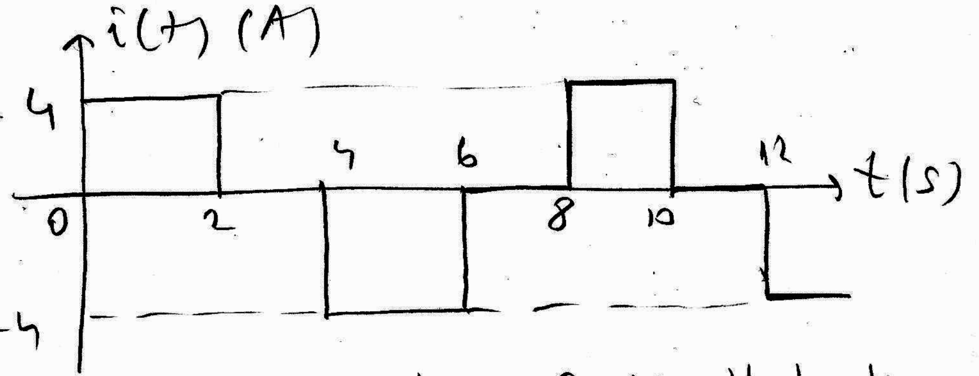
c)



$P = \frac{V_{rms}^2}{R} = \frac{1,885^2}{2}$

$\boxed{P = 1,776\,W}$

2



Dalga şekli verilen akım R=10Ω'luk dirençten geçerse direncin gücü P=?

Çözüm

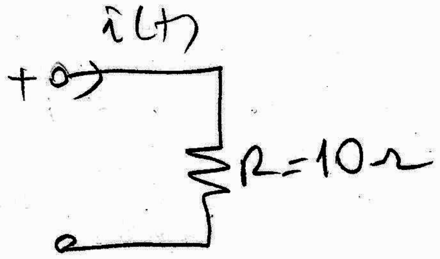
$P = R \cdot I_{rms}^2 \Rightarrow I_{rms} = ?$ T = 8s (şekilden)

$$I_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T u^2(t)\,dt}$$

$$I_{rms} = \sqrt{\frac{1}{8} \cdot \left[\int_0^2 4^2 \cdot dt + \int_4^6 (-4)^2 dt \right]}$$

$$I_{rms} = \sqrt{\frac{1}{8} \left[16t \Big|_0^2 + 16t \Big|_4^6 \right]} = \sqrt{\frac{1}{8} \left[16(2-0) + 16(6-4) \right]}$$

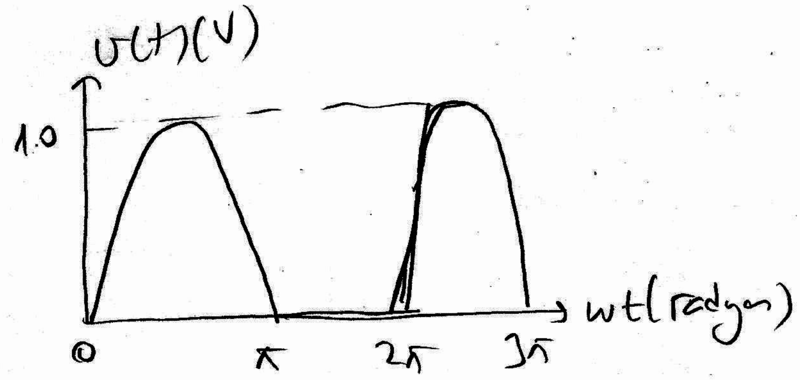
$$I_{rms} = \sqrt{\frac{1}{8} \cdot 64} \Rightarrow \boxed{I_{rms} = 2,828\,A}$$

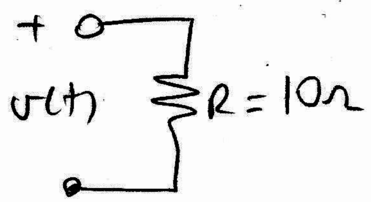


$$P = R\, I_{rms}^2 = 10 \cdot 2,828^2$$

$$\boxed{P = 80W}$$

(4)





a) $V_{ort}=?$, b) $V_{rms}=?$, c) $P=?$

Çözüm

$$v(wt)=\begin{cases}10.\sin(wt) & 0\leq wt\leq\pi\\ 0 & \pi\leq wt\leq 2\pi\end{cases}$$

a) $V_{ort}=\frac{1}{T}\cdot\int_0^T v(t).dt$ veya $V_{ort}=\frac{1}{2\pi}\int_0^{2\pi} v(wt)d(wt)$

$$V_{ort}=\frac{1}{2\pi}\left[\int_{wt=0}^{\pi}10\sin(wt)dwt+\cancel{\int_{\pi}^{2\pi}0.d(wt)}\right]$$

$wt=x$ diyelim (basit görünmesi için)

$$V_{ort}=\frac{1}{2\pi}\left[\int_{x=0}^{\pi}10\sin x dx\right]=\frac{1}{2\pi}.10.(-\cos x)\Big|_{x=0}^{\pi}$$

$$V_{ort}=\frac{10}{2\pi}(-\cos\pi+\cos 0)=\frac{10}{2\pi}.2\Rightarrow\boxed{V_{ort}=\frac{10}{\pi}=3,183V}$$

b) $V_{rms}=?$

$V_{rms}=\sqrt{\frac{1}{T}\int_{t=0}^{T}v^2(t)dt}$ veya $V_{rms}=\sqrt{\frac{1}{2\pi}\int_{wt=0}^{2\pi}v^2(wt)d(wt)}$

$V_{rms}=V$ diyelim $P=\frac{V^2}{R}$ olur

$$V = \sqrt{\frac{1}{2\pi}\left[\int_{wt=0}^{\pi} (10 \cdot \sin(wt))^2 \cdot d(wt)\right]}$$

$wt = x$ diyelim

$$V = \sqrt{\frac{1}{2\pi} \cdot \int_{x=0}^{\pi} 100 \sin^2 x \, dx}$$

$$\sin^2 x = \frac{1 \mp \cos 2x}{2}$$

Hatırlatma

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(2a) = \cos^2 a - \sin^2 a$

$\cos^2 a = 1 - \sin^2 a$

$\cos 2a = 1 - 2\sin^2 a$

$\boxed{\sin^2 a = \frac{1 \mp \cos 2a}{2}}$

$$V = \sqrt{\frac{1}{2\pi} \int_0^{\pi} 100 \cdot \left(\frac{1 \mp \cos 2x}{2}\right) dx}$$

$$V = \sqrt{\frac{25}{\pi} \cdot \int_0^{\pi} (1 + \cos 2x) dx} = \sqrt{\frac{25}{\pi}\left[\int_0^{\pi} dx + \int_0^{\pi} \cos 2x \, dx\right]}$$

$$V = \sqrt{\frac{25}{\pi}\left[x\Big|_0^{\pi} + \frac{1}{2}\sin 2x\Big|_0^{\pi}\right]} = \sqrt{\frac{25}{\pi}\left[(\pi - 0) + \frac{1}{2}(\underbrace{\sin 2\pi - \sin 0}_{0})\right]}$$

$$V = \sqrt{\frac{25\cancel{\pi}}{\cancel{\pi}}} \Rightarrow \boxed{V = 5 \text{ Volt}}$$

c) $P = \frac{V^2}{R} = \frac{5^2}{10} \Rightarrow \boxed{P = 2{,}5 W}$