MACHINE ELEMENT II, FORMULA SHEET
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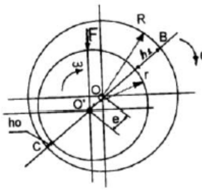
BEARINGS

Radial Sliding Bearings



R : bearing radius, r : shaft radius
Radial clearance $c = R - r$
Exantrisity : $e = \overline{OO'}$

h_1 max. film thichness and ve h_o min. film thichness
$h_o = h_{min} = c - e$, $h_{max} = h_1 = c + e$
eksantrisite oranı, $\varepsilon = e / c$
relatif yatak boşluğu (boyutsuz boşluk) $\psi = c / R$

Average pressure , $p_m = F / (L.D)$ (D bearing diameter)

Sommerfold number $So = p_m.\psi^2 / (\eta . \omega_{geçiş})$, $\omega_{geçiş} = \pi \, n_{geçiş}/30$

Viscosity , η [$Nsn / m^2 = 10^3$ c P] , $\omega_{geçiş}$ [1 / sn], p_m (N/ m²)

Lenght /diameter ratio: $L / D = 0,5 \div 1,5$, $L / D = 1$ is a good choise.

Dimensionless clearance $\psi \cong 0,0008 \sqrt[4]{U}$ (for bronze and white metal) (U : m/s)

$\psi = 0,004$ for polymeric materials.

Table 1. $1 / So$ and μ / ψ for different ε

	ε	0,95	0,9	0,8	0,7	0,6
L / D =1 için	1 / So	0,054	0,12	0,28	0,48	0,75
	μ / ψ	0,675	1,06	1,71	2,36	3,21
L / D =1/2 için	1 / So	0,075	0,196	0,577	1,16	2,01
	μ / ψ	0,869	1,59	3,25	5,48	8,08

ROLLING BEARINGS

Equivalent force, $F_{eş} = x.F_r + y.F_a$
F_r : radial force , x: Radial factor, F_a : axial force, y: Axial factor

X and y are determined from table according to Fa/Fr ratio.

$L = (\frac{C}{F_{eş}})^p$, L : life as million revolution, C : dynamic load factor

p: Life coeficient p = 3 (ball bearings), p = 10/3 (roller bearing)

Life as an hour $L_h = L10^6 / 60.n$

COUPLINGS

Coupling moment:
$M_k = M_{sür} = SM_d$ (friction based couplings);

Rigid couplings

Clamped (compression) coupling, $M_k \geq M_d$
Friction moment, $M_k = M_{sür} = (\pi/4)\mu d^2 L p_i$ $\alpha \cong 2° \div 3°$, Sleeve length, $L \cong (3 \div 4)d$

Force to assemble a ring : $Fa = \pi D_o b p_a (tg\alpha + \mu)$

Flange coupling

$Da = 4d$, $D_o = 3d$, $Di = 2d$, $L = 1,5d$, d: shaft diameter
$M_k = M_s = M_k = (2/3).\pi \, \mu. \, p. \, (R_a^3 - R_i^3)$, R_a: flange outer radious, R_i: Flange inner radious

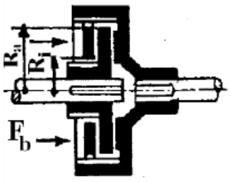
pressure: $p = \dfrac{F_{ön} n}{\pi (R_a^2 - R_i^2)}$ n: number of bolts, Fön: Preload for one bolt

Control oft he bolts for shear: $\tau = \dfrac{F_ç}{n(\pi . d_1^2 / 4)} \leq \tau_{em}$ d_1: minor diameter of bolt, $\tau_{em} = \sigma_{em} / 2$, $\sigma_{em} = 0,6\sigma_{Ak}$

Circumferantial force, $F_ç = M_d / (D_o/2)$

Clutches:

Basic disc clutch: $p = F_b / [\pi(R_a^2 - R_i^2)]$



$M_k = M_{sür} = (2/3).\pi\ \mu.\ p.\ (R_a^3 - R_i^3)$,

$M_k = (2/3).F_b.\ \mu\ (R_a^3 - R_i^3) / (R_a^2 - R_i^2)$

$R_m = (2/3)\ (R_a^3 - R_i^3)/(R_a^2 - R_i^2)$, $M_k = \mu.F_b.R_m$

Cone clutches

Friction moment: $M_{sür} = (2/3).\pi\ \mu.\ p.\ (R_a^3 - R_i^3)/\mathrm{Sin}\alpha$

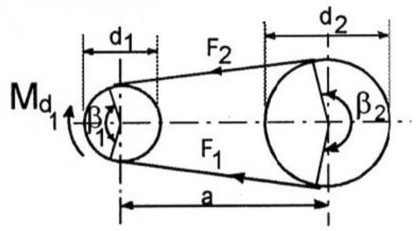
Assembly force: $F_b = F_n \sin\alpha$ Pressure: $$p = \frac{F_n \mathrm{Sin}\,\alpha}{\pi(R_a^2 - R_i^2)}$$ $\alpha = 12° \div 15°$

$M_k = \mu.F_n R_m$

For multiple friction surface clutches:

$M_k = i.\mu.F_n R_m$ i: number of friction surface.

Belt and Pulley Mechanicsm



Kullanılan Semboller:

b : Belt width [mm]

L :Belt length [mm]

A : Belt cross section area [mm²]

s : belt thickness [mm]

d : Pulley diameter [mm]

B : Pulley width [mm]

F_{mr} : Centrifugal force [N]

F_1, F_2 : Forces [N]

β_1, β_2 : wrap angles [°, rad]

P : Power [kw]

α : Pulley angle [°]

M_d : Torque [Nm]

i : speed ratio = d_2 / d_1

$\beta = \beta_1$ $\beta_2 = 2\pi - \beta_1$ $$\cos\beta/2 = \frac{d_2 - d_1}{2.a}$$ $L = \beta d_1/2 + [(2.\pi - \beta)d_2/2] + 2.a\mathrm{Sin}\ \beta/2$

$F_1 - F_2 = M_d / r = F_ç$ $$Fr = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\mathrm{Cos}\beta}$$

$F_1 / F_2 = e^{\mu\beta}$, $\sigma_1 / \sigma_2 = e^{\mu\beta}$

Power: $P = F_ç.v$

Centrifugal force $F_{mç} = A.v^2.\rho$

Stress due to centrifugal force $\sigma_{mç} = F_{mç}/A$

ρ: Density of belt material, A: Belt cross section area

Bending stress : $\sigma_e = E_e\varepsilon$ strain : $\varepsilon = s/d$

Total stress: $\sigma_{top} = \sigma_1 + \sigma_{mç} + \sigma_e \le \sigma_{em}$

$\sigma_{em} = \sigma_K / S$, σ_K : Ultimate strength of belt material, S: Safety factor

B is found from strength : $\sigma_1 = F_1/(b.s) \le \sigma_{em}$ (bending and centrifugal stress are neglected)

Number of pulley: $z_k = P.C_2 / (P_1.C_1.C_3)$ P_1 : The power that one selected belt can transmit.

Gear Mechanism

Module (m)	$m = t / \pi \quad (= d / z)$
Number of teeth (z)	$z = do / m$
Pitch (t)	$t = \pi m$
Pitch diameter (do)	$do = z m$
Addendum diameter (d_b)	$d_b = d + 2h_b = m (z + 2)$
Dedendum diameter (d_{ta})	$d_{ta} = d - 2h_t = m (z - 2{,}5)$
Pressure angle (α)	$\alpha = 20°$
Base circle diameter (d_t)	$d_t = do \cos\alpha$
Speed (v)	$v = \pi\, do\, n / 60 \quad [m / s]$
Speed ratio (i)	$i_{12} = \omega_1 / \omega_2 = n_1 / n_2 = d_{o2} / d_{o1} = z_2 / z_1$
Gear width (b)	$b = \psi_m m$ ψ_m : Width number according to module $\psi_d = b/d_o$ (According to pitch diameter) $\psi_t = b/t = b/(\pi.m)$ (According to pitch)
Total speed ratio: i_{top}	$i_{top} = i_{12}.i_{34}$ (for 2 stages)
Distance between shafts, a	$a = (d_{o1}+d_{o2})/2$

Module according to strength

$F_ç = S.M_d / (do /2)$ $\qquad \sigma_{em} = \sigma_D / K_ç$

$$m = \sqrt[3]{\frac{2.S.M_d K_d .K_f}{z.\psi_m .\varepsilon.\sigma_{em}}},$$

$Md = 9550.P/n$ (Nm)

Module according to contact pressure

$$m = \sqrt[3]{\frac{2.S.M_d .E.K_d}{z^2 .p_{em}^2 .\varepsilon.\psi_m} \cdot \frac{i+1}{i}}$$

$$\frac{1}{E} = \frac{1}{2}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$ E : equivalent elasticity modulus

$p_{em} = 0{,}7.\sigma_K$, $\sigma_K \approx 0{,}35\ H_B$

In control calculations:

Strength

$$\sigma_{eş} = \frac{F_ç .K_f}{b.m} \le \frac{\sigma_D}{K_ç K_d K_\varepsilon} \quad ;$$

Contact pressure

$$p_{max} = K_m .K_\alpha .K_\varepsilon \sqrt{\frac{K_d .F_ç}{bd_o} \cdot \frac{i+1}{i}} \le p_{em}$$

$K_ç$: Notch factor, K_d: Dynamic load factor, K_f :form factor

Material factor, $K_m = \sqrt{0{,}35.E}$ (E_1=E_2=E). If $E_1 \neq E_2$, $K_m = \sqrt{0{,}35.\frac{2.E_1 .E_2}{E_1 + E_2}}$ $\qquad K_\alpha$=1,76, $\quad K_\varepsilon = \frac{1}{\varepsilon}$.

Forces:

Gear force: $F_z = F_ç / \cos\alpha$ **radial force: $F_r = F_ç . tg\alpha$** **Circumferential force: $F_ç = M_d$ / (do /2)**

HELICAL GEARS

Transverse module (m_a), Normal module (m_n)	$m_a = t_a / \pi = m_n / \cos\beta$
Pitch circle diameter, d_a	$d_a = m_a z$
Distance b/w shaft axiles, a	$a = (z1 + z2)m_a / 2$
Addendum diameter , (d_b), Dedendum diameter, (d_{ta})	$db = z.m_a + 2.m_n$ $d_{ta} = z.m_a - 2{,}5.m_n$
Base circle diameter, dt	$dt = da.\cos\alpha_a$
Transverse pressure angle (α_a), Normal pressure angle (α_n)	$tg\alpha_a = tg\ \alpha_n / \cos\beta$ $\alpha_n = 20°$
Gear width b	$b = \psi d.d_a$, $b = \psi_m.m_n$
Equivalent number of teeth (z_n)	$z_n = z / \cos^3\beta$

Forces:

$F_ç = 2.S.M_d / d_a$ $F_r = F_Z.\sin\alpha_n$

$F_n = F_Z.\cos\alpha_n$ $F_a = F_n.\sin\beta = F_Z.\cos\alpha_n.\sin\beta = F_ç.tg\beta$

Module for strenght consideration:

$$m_n = \sqrt[3]{\frac{2.S.M_d.K_d.K_{fn}.\cos\beta}{z.\psi_m.\varepsilon.\sigma_{em}}} \quad .\ (\sigma_{em}=\sigma_D/K_ç)$$

$\beta = 10° \div 45°$

K_d : Dynamic load factor, K_{fn}: Form factor

Module for contact pressure:

$$m_n = \sqrt[3]{\frac{2.S.M_d.E.K_d.\cos^4\beta}{z_1^2.p_{em}^2.\varepsilon.\psi_m}\cdot\frac{i+1}{i}}$$

Control calculations:

$$\sigma_{eş} = K_d.K_{fn}.\frac{F_ç}{m_n.\varepsilon.b} \le \sigma_{em}$$

$$p_{max} = K_m.K_\alpha.K_\varepsilon.K_\beta\sqrt{\frac{K_d.F_ç}{b.d_a}\cdot\frac{i+1}{i}}$$