

Sorui $y'' = 2y^3 + 8y$ denkleminin $x = \frac{\pi}{4}$

icin $y=2$, $y'=8$ olan özel çözümlerini bulunuz.

x-yok.

$$y' = p$$

$$y'' = p \frac{dp}{dy}$$

$$p \frac{dp}{dy} = 2y^3 + 8y$$

$$\int p dp = \int (2y^3 + 8y) dy$$

$$\frac{p^2}{2} = \frac{1}{2} y^4 + 4y^2 + C_1$$

$$p^2 = y^4 + 8y^2 + 2C_1$$

$$p = \pm \sqrt{y^4 + 8y^2 + 2C_1}$$

$$\frac{dy}{dx} = \pm \sqrt{y^4 + 8y^2 + 2C_1}$$

$$8 = \pm \sqrt{16 + 32 + 2C_1}$$

$$64 = 48 + 2C_1$$

$$16 = 2C_1$$

$$\boxed{C_1 = 8}$$

$$\frac{dy}{dx} = \pm \sqrt{y^4 + 8y^2 + 16}$$
$$= \pm \sqrt{(y^2 + 4)^2}$$

$$\frac{dy}{dx} = \pm (y^2 + 4) \Rightarrow \pm \frac{dy}{y^2 + 4} = \int dx$$

$$\pm \frac{1}{2} \arctan \frac{y}{2} = x + C_2$$

$$x = \frac{\sqrt{17}}{4} \text{ için } y = 2, \quad y' = 8$$

$$\pm \frac{1}{2} \operatorname{arctan} u = \frac{\sqrt{17}}{4} + c_2$$

$$\pm \frac{\sqrt{17}}{8} = \frac{\sqrt{17}}{4} + c_2$$

$$i) \quad \frac{\sqrt{17}}{8} = \frac{\sqrt{17}}{4} + c_2 \Rightarrow c_2 = -\frac{\sqrt{17}}{8}$$

$$ii) \quad -\frac{\sqrt{17}}{8} = \frac{\sqrt{17}}{4} + c_2 \Rightarrow c_2 = -\frac{\sqrt{17}}{8} - \frac{\sqrt{17}}{4} = -\frac{3\sqrt{17}}{8}$$

$$\otimes \quad \frac{1}{2} \operatorname{arctan} \frac{y}{2} = x - \frac{\pi}{8}$$

$$\boxed{\operatorname{arctan} \frac{y}{2} = 2\left(x + \frac{\pi}{8}\right)}$$

$$\otimes \quad \frac{1}{2} \operatorname{arctan} \frac{y}{2} = x - \frac{3\pi}{8}$$

$$\boxed{\operatorname{arctan} \frac{y}{2} = 2\left(x - \frac{3\pi}{8}\right)}$$