GEOMETRIC CONTROL STRUCTURES ON $SE(3)$ AND ITS COSET SPACES WITH KINEMATIC PERSPECTIVE

Okan DUMAN

*D*epartment of Mathematics, Yildiz Technical University, Istanbul, Türkiye

Corresponding Author's E-mail: oduman@yildiz.edu.tr

ABSTRACT

The present research is situated at the intersection of geometric robotics and the theory of control systems on homogeneous spaces endowed with Riemannian (or sub-Riemannian) structures. The work is motivated by a fundamental structure in robotic kinematics: the forward kinematics map, which projects the state space of a manipulator's joints onto the state space of its end-effector, the latter often being the Special Euclidean group $SE(3)$. For redundant systems, this projection possesses a rich geometric structure, intimately related to the characterization of internal motions and cyclic behaviors. While this framework is well-developed for systems with commutative state spaces, its extension to more general, non-commutative settings, where the underlying symmetry is that of a Lie group, remains an area ripe for deeper exploration. This study proposes to systematically address this generalization by investigating the natural projection of dynamical systems from a Lie group onto its associated homogeneous (coset) manifold spaces. We will employ the formalism of linear control systems on these quotient spaces to model and analyze the induced dynamics. Our approach involves a detailed geometric study of such systems, with $SE(3)$ serving as a canonical and physically motivated example, to classify the resulting controllability properties and invariant structures. The ultimate aim is to develop a unifying geometric perspective that can offer fresh insights into the motion planning and analysis of complex mechanical systems, translating classical robotic concepts into the language of invariant distributions and a rigorous differential geometric perspective.

***K*eywords** Homogeneous spaces · sub-Riemannian manifolds · Controllability

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