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ON THE NUMERICAL SOLUTION OF LANE-EMDEN DIFFERENTIAL EQUATIONS WITH INDEX-m

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Abstract

In this paper, numerical solution of Lane-Emden differential equations with index-m has been presented by Padé Approximation. However, in general, the solution of Lane-Emden equation cannot be given analytically. This type of non-linear differential equations can be solved only numerically. An example was presented to show the ability of the method for Lane-Emden differential equation. Numerical solution to Lane-Emden differential equations with index-m has been discussed. MAPLE computer algebra systems are used for numerical calculations [9].

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1. Introduction

Many problems in the literature for example, mathematical physics, astrophysics and celestial mechanics such as theory of stellar structure, the thermal behavior of a spherical cloud of gas, isothermal gas spheres and thermionic currents can be formulated as the so-called *Lane-Emden* type equation [8, 11, 12] defined in the form

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+f(y)=0, \tag{1.1a}$$

subject to boundary condition

$$y(0)=y_0,\quad y'(0)=0,\quad 0<x<\infty, \tag{1.1b}$$

where $f(y)$ is some given non-linear function of $y(x)$ and y_0 is a constant. It is well known that an *analytic solution* of Lane-Emden type equation (1.1) is always possible in the neighborhood of the singular point $x=0$ for the boundary conditions $y(0)=y_0,\ y'(0)=0$ [13]. In the literature, Lane-Emden type equation has applications. For example, it was used to model the thermal behavior of spherical cloud of gas acting under mutual attraction of its molecules [11, 13] and subject to the classical laws of thermodynamics when $f(y)=y^m$, where m is constant and it has index-m . A second-order ordinary differential equation arising in the study of stellar interiors, also called the *polytrophic differential equations*. It is given by

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+y^m=0, \tag{1.2}$$

$$\frac{1}{x^2}\left(2x\frac{dy}{dx}+x^2\frac{d^2y}{dx^2}\right)+y^m=\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+y^m=0, \tag{1.3}$$

[14, 15]. It has the boundary conditions

$$y(0)=1, \tag{1.4}$$

$$\left[\frac{dy}{dx}\right]_{x=0}=0. \tag{1.5}$$

The Lane-Emden equation can be solved analytically for values of

index $m = 0, 1$ and 5. For other values of m, The Lane-Emden equation has to be solved numerically [8, 14]. For $m = 0,$ Lane-Emden differential equation [8] is

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+1=0, \tag{1.6}$$

directly solving gives

$$\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)=-x^2, \tag{1.7}$$

$$\int d\left(x^2\frac{dy}{dx}\right)=-\int x^2dx, \tag{1.8}$$

$$y(x)=y_0-c_1x^{-1}-\frac{1}{6}x^2. \tag{1.9}$$

The boundary condition $y(0) = 1,$ then gives $y_0 = 1$ and $c_1 = 0,$ so the

$$y(x)=1-\frac{1}{6}x^2, \tag{1.10}$$

and $y(x)$ is parabolic.

For $m = 1,$ Lane-Emden differential equation [8] is becomes

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+y=0, \tag{1.11}$$

$$\frac{d}{dx}\left(x^2\frac{dy}{dx}\right)+x^2y=0, \tag{1.12}$$

which is the spherical Bessel differential equation

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+[k^2r^2-m(m+1)]R=0, \tag{1.13}$$

with $k = 1$ and $n = 0,$ so the solution is

$$y(x)=Aj_0(x)+Bn_0(x). \tag{1.14}$$

The boundary condition $y(0) = 1$ gives

$$y(x)=j_0(x)=\frac{\sin x}{x}, \tag{1.15}$$

where $j_0(x)$ is a spherical Bessel function of the first kind [8, 14].

For $m = 5$, Applying Emden's transformation [14], the Lane-Emden equation becomes

$$\frac{d^2z}{dt^2} = \frac{1}{4}z(1 - z^4), \tag{1.16}$$

and finally,

$$y(x) = \left(1 + \frac{1}{3}x^2\right)^{-\frac{1}{2}}. \tag{1.17}$$

It is well known that an *analytic* solution of Lane-Emden type equation (1.1) is always possible [13] in the neighborhood of the singular point $x = 0$ for the boundary conditions $y(0) = y_0$, $y'(0) = 0$.

There is another interesting form of $f(y)$ given by $f(y) = (y^2 - c)^{2/3}$. The Lane-Emden equation with $f(y) = (y^2 - c)^{2/3}$ gives the *white-dwarf* equation introduced by Chandrasekhar in his study of gravitational potential of the degenerate white-dwarf stars [13]. The other non-linear forms of $f(y)$ are exponential functions $f(y) = e^{\pm y(x)}$. The model describes in the isothermal gas spheres and the theory of thermionic currents [11].

In the present paper, we consider an initial-value problem which is a generalization of (1.1)

$$y''(x) + \frac{n}{x}y'(x) = -[y(x)]^m, \quad m \geq 0, \quad n \geq 0, \tag{1.18a}$$

$$y(0) = 1, \quad y'(0) = 0, \quad 0 < x < \infty, \tag{1.18b}$$

where $m \geq 0$, $n \geq 0$, $x > 0$ and $\frac{n}{x}$ is singular.

Many physical problems are most easily initially modeled as a system of differential-algebraic equations (DAEs) [5]. Some numerical methods have been developed, using both BDF [1, 3, 5, 6, 10] and implicit Runge-Kutta methods [2, 10]. These methods are only directly suitable for low index problems and often require that the problem to have special structure. Although many important applications can be solved by these methods there is a need for more general approaches.

The most general form of a DAE is given by

$$F(t,\ y,\ y') = 0, \tag{1.19}$$

where $\partial F/\partial y'$ may be singular. The rank and structure of this Jacobian matrix may depend, in general, on the solution $y(t)$ and for simplicity we will always assume that it is independent of t. The important special case of a semi-explicit DAE, or an ODE with constraints,

$$x' = f(t,\ x,\ z), \tag{1.20a}$$

$$0 = g(t,\ x,\ z). \tag{1.20b}$$

This is a special case of (1.19). The index is 1, if $\partial g/\partial z$ is nonsingular, because then one differentiation of (1.20b) yields z' in principle. For the semi-explicit index-1 DAE, we can distinguish between differential variables $x(t)$ and algebraic variables $z(t)$. The algebraic variables may be less smooth than the differential variables by one derivative. In the general case, each component of y may contain a mix of differential and algebraic components, which makes the numerical solution of such high-index problems much harder and riskier. The semi-explicit form is decoupled in this sense. On the other hand, any DAE (1.19) can be written in the semi-explicit form (1.20) but with the index increased by 1, upon defining $y' = z$, which gives

$$y' = z, \tag{1.21a}$$

$$0 = F(t,\ y,\ z). \tag{1.21b}$$

Needless to say, this rewriting alone does not make the problem easier to solve. The converse transformation is also possible, given a semi-explicit index-2 DAE system (1.20), let $w' = z$. It is easily shown that the system

$$x' = f(t,\ x,\ w'), \tag{1.22a}$$

$$0 = g(t,\ x,\ w'). \tag{1.22b}$$

is an index-1 DAE and yields exactly the same solution for x as (1.20). The classes of fully implicit index-1 DAEs of the form (1.19) and semi-explicit index-2 DAEs of the form (1.20) are therefore equivalent. We define the index of a DAE. For general DAE systems (1.19), the index along a solution $y(t)$ is the minimum number of differentiations of the

system which would be required to solve for y' uniquely in terms of y and t (i.e., to define an ODE for). Thus, the index is defined in terms of the over determined system

$$F(t,\ y,\ y') = 0,$$

$$\frac{dF}{dt}(t,\ y,\ y',\ y'') = 0,$$

$$\vdots$$

$$\frac{d^p F}{dt^p}(t,\ y,\ y',\ ...,\ y^{(p+1)}) = 0, \tag{1.23}$$

to be the smallest integer p so that y' in (1.23) can be solved for in terms of y and t. We note that in practice, differentiation of the system as in (1.23) is rarely done in a computation. However, such a definition is very useful in understanding the underlying mathematical structure of the DAE system, and hence is selecting an appropriate numerical method [4, 7].

2. Special Differential-Algebraic Equations (DAE) Forms

Most of the higher-index problems encountered in practice can be expressed as a combination of more restrictive structures of ODEs coupled with constraints. In such systems, the algebraic and differential variables are explicitly identified for higher-index DAEs as well, and the algebraic variables may all be eliminated using the same number of differentiation. These are called *Hessenberg forms* of the DAE and are given below.

2.1. Hessenberg index-1

$$x' = f(t,\ x,\ z), \tag{2.1a}$$

$$0 = g(t,\ x,\ z). \tag{2.1b}$$

Here the Jacobian matrix function g_z is assumed to be nonsingular for all t. This is also often referred to as a semi-explicit index-1 system. Semi-explicit index-1 DAEs are very closely related to implicit ODEs. Using the implicit function theorem, we can in principle solve for z in

(2.1b). Substituting z into (2.1a) yields an ODE in x. For various reasons, this procedure is not always recommended for numerical solution.

2.2. Hessenberg index-2

$$x' = f(t, x, z), \tag{2.2a}$$

$$0 = g(t, x). \tag{2.2b}$$

Here the product of Jacobians $g_x f_z$ is nonsingular for all t. Note the absence of the algebraic variables z from the constraints (2.2b). This is a pure index-2 DAE, and all algebraic variables play the role of index-2 variables.

2.3. Hessenberg index-3

$$x' = f(t, x, y, z), \tag{2.3a}$$

$$y' = g(t, x, y), \tag{2.3b}$$

$$0 = h(t, y). \tag{2.3c}$$

Here the product of three matrix functions $h_y g_x f_z$ is nonsingular.

The index of a Hessenberg DAE is found, as in the general case, by differentiation. However, here only algebraic constraints must be differentiated [2].

3. The Method

Consider the differential-algebraic equation (DAE)

$$F(y, y', t) = 0, \tag{3.1}$$

with initial condition

$$y(x_0) = y_0,$$

where and F and y are vector functions. The solutions of (3.1) can be assumed that

$$y = y_0 + ex, \tag{3.2}$$

where e is a vector function. Substitute (3.2) into (3.1) and neglect higher order terms. We have the linear equation of e in the form

$$Ae = B, \tag{3.3}$$

where A and B are constant matrixes. Solving this (3.3), the coefficients of e in (3.2) can be determined. Repeating the above procedure for higher terms, we can get the arbitrary order power series of the solutions for (3.1). The power series given by the above procedure can be transformed into Padé series and we have numerical solution of DAE in (3.1).

4. Power Series of Solution for DAEs

We define another type of power series in the form

$$f(x) = f_0 + f_1 x + f_2 x^2 + \cdots + (f_n + p_1 e_1 + \cdots + p_m e_m) x^n, \tag{4.1}$$

where $p_1, p_2, ..., p_m$ are constants. $e_1, e_2, ..., e_m$ are bases of vector, m is the size of vector e. y is a vector with m elements in (3.2). Every element can be represented by the power series in (4.1).

$$y_i = y_{i,0} + y_{i,1} x + y_{i,2} x^2 + \cdots + e_i x^n, \tag{4.2}$$

where y_i is the ith element of y. Substituting (4.2) into (3.1), we can get the following:

$$f_i = (f_{i,n} + p_{i,1} e_1 + \cdots + p_{i,m} e_m) x^{n-j} + O(x^{n-j+1}), \tag{4.3}$$

where f_i is the ith element of $f(y, y', x)$ in (3.1) and j is 0 if $f(y, y', x)$ have y', 1 if do not. From (4.3) and (3.3), we can determine the linear equation in (3.3) as follows:

$$A_{i,j} = P_{i,j}, \tag{4.4a}$$

$$B_i = -f_{i,n}. \tag{4.4b}$$

Solving this linear equation, we have $e_i (i = 1, ..., m)$. Substituting e_i into (4.2), we have $y_i (i = 1, ..., m)$, which are polynomials of degree n. Repeating this procedure from (4.4), we can get the arbitrary order power series of the solution for DAEs in (3.1). If we repeat the above procedure, then we have numerical solution of DAEs in (3.1) [6, 7].

5. Padé Series

The Power series can be transformed into Padé series easily. Padé series is defined in the following:

$$a_0 + a_1x + a_2x^2 + \cdots = \frac{p_0 + p_1x + \cdots + p_Mx^M}{1 + q_1x + \cdots + q_Lx^L}. \tag{5.1}$$

Multiply both sides of (5.1) by the denominator of right-hand side in (5.1) and compare the coefficients of both sides in (5.1). We have

$$a_l + \sum_{k=1}^{M} a_{l-k}q_k = p_l \quad (l = 0, ..., M), \tag{5.2}$$

$$a_l + \sum_{k=1}^{L} a_{l-k}q_k = 0 \quad (l = M + 1, ..., M + L). \tag{5.3}$$

Solving the linear equation in (5.3), we have $q_k(k = 1, ..., L)$. And substituting q_k into (5.1), we have $p_k(l = 0, ..., M)$ [6].

6. An Example

We will examine more closely Lane-Emden like equations. If we replace the standard coefficient $\frac{2}{x}$ of y' in Lane-Emden equation by $\frac{n}{x}$, for real n, $n \geq 0$, then we write down Lane-Emden like equation in general fashion as

$$y'' + \frac{n}{x}y' + f(y) = 0, \tag{6.1}$$

with boundary conditions given by

$$y(0) = \alpha, \quad y'(0) = 0, \tag{6.2}$$

for $n = 0$, the boundary $x = 0$ is ordinary point, whereas for $n \neq 0$, the boundary $x = 0$ is singular point [13]. In the following, we will examine by considering $f(y) = y^m$, $m \geq 0$. we consider

$$y'' + \frac{n}{x}y' + y^m = 0 \text{ for real } n, m;\ n \geq 0, m \geq 0, \tag{6.3}$$

subject to the boundary conditions, which are of most interest?

$$y(0) = 1 \quad y'(0) = 0. \tag{6.4}$$

If we define $y_1 = y$, $y_2 = y_1'$, then we have the following differential equation system

$$\begin{aligned} y_1' &= y_2, \\ y_2' &= -\frac{n}{x} y_2 - y_1^m, \end{aligned} \tag{6.5}$$

subject to boundary conditions

$$y_1(0) = 1, \quad y_2(0) = 0.$$

From boundary condition, solutions of (6.5) can be supposed as

$$\begin{aligned} y_1(x) &= y_{0,1} + e_1 x \Rightarrow y_1(x) = 1 + e_1 x, \\ y_2(t) &= y_{0,2} + e_2 t \Rightarrow y_2(t) = 0 + e_2 x. \end{aligned} \tag{6.6}$$

Substituting (6.6) into (6.5) and neglect higher order terms, we have

$$e_1 + Q(x) = 0,$$

$$(n+1)e_2 + 1 + Q(x) = 0, \tag{6.7}$$

$$Ae = B, \tag{6.8}$$

where

$$A = \begin{pmatrix} 1 & 0 \\ 0 & (n+1) \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \; e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}.$$

From equation (6.4), we have linear equation

$$\begin{pmatrix} 1 & 0 \\ 0 & (n+1) \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

Solving this linear equation system, we have

$$e = \begin{pmatrix} 0 \\ -\dfrac{1}{(n+1)} \end{pmatrix},$$

and

$$y_1(x) = 1, \quad y_2(x) = 1 - \frac{1}{n+1} x, \tag{6.9}$$

From (6.9) the solutions of (6.5) can be supposed as

$$y_1(x) = 1 + e_1 x^2,$$

$$y_2(t) = 1 - \frac{1}{(n+1)} x + e_2 x^2. \tag{6.10}$$

In like manner, substitute (6.10) into (6.5) and neglect higher order terms, then we have

$$\left(\frac{1}{(n+1)} + 2e_1\right)x + Q(x^2) = 0,$$

$$((2+n)e_2)x + Q(x^2) = 0, \tag{6.11}$$

where

$$A = \begin{pmatrix} 2 & 0 \\ 0 & (2+n) \end{pmatrix}, \quad B = \begin{pmatrix} -\frac{1}{n+1} \\ 0 \end{pmatrix}, \quad e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}.$$

From equation (6.11), we have linear equation

$$\begin{pmatrix} 2 & 0 \\ 0 & (2+n) \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{n+1} \\ 0 \end{pmatrix}. \tag{6.12}$$

Solving this linear equation, we have

$$e = \begin{pmatrix} -\frac{1}{2(n+1)} \\ 0 \end{pmatrix},$$

therefore,

$$y_1(x) = 1 - \frac{1}{2(n+1)} x^2,$$

$$y_2(t) = 1 - \frac{1}{(n+1)} x. \tag{6.13}$$

From (6.13) the solutions of (6.5) can be supposed as

$$y_1(x) = 1 - \frac{1}{2(n+1)} x^2 + e_1 x^3,$$

$$y_2(t) = 1 - \frac{1}{(n+1)} x + e_2 x^3. \tag{6.14}$$

Repeating above procedure, we have the power series solutions of $y_1(x)$ and $y_2(x)$ for real values of $n \geq 0$ and $m \geq 0$ in the following forms

$$y_1(x) = 1 - \frac{1}{2(n+1)}x^2 + \frac{m}{8(n+1)(n+3)}x^4 - \frac{(2n+4)m^2 - (n+3)m}{48(n+1)^2(n+3)(n+5)}x^6$$

$$+ \frac{(6n^2+32n+34)m^3 - (7n^2+46n+63)m^2 + (2n^2+16n+30)m}{384(n+1)^3(n+3)(n+5)(n+7)}x^8 + \cdots,$$

$$y_2(x) = 1 - \frac{1}{(n+1)}x + \frac{m}{4(n+1)(n+3)}x^3 - \frac{(2n+4)m^2 - (n+3)m}{8(n+1)^2(n+3)(n+5)}x^5$$

$$+ \frac{(6n^2+32n+34)m^3 - (7n^2+46n+63)m^2 + (2n^2+16n+30)m}{48(n+1)^3(n+3)(n+5)(n+7)}x^7 + \cdots. \quad (6.15)$$

We have obtained the power series solution $y(x)$ of Lane-Emden like equation (6.3) for real values of $n \geq 0$ and $m \geq 0$ from $y_1(x)$ in the form

$$y(x) = 1 - \frac{1}{2(n+1)}x^2 + \frac{m}{8(n+1)(n+3)}x^4 - \frac{(2n+4)m^2 - (n+3)m}{48(n+1)^2(n+3)(n+5)}x^6$$

$$+ \frac{(6n^2+32n+34)m^3 - (7n^2+46n+63)m^2 + (2n^2+16n+30)m}{384(n+1)^3(n+3)(n+5)(n+7)}x^8 + \cdots \quad (6.16)$$

Padé $y(x)$ is

$$p = [4/3] = 1/48\,(288n^4 + 2880n^3 + 9216n^2 + 10944n + 4320$$

$$+ (96mn^3 + 576mn^2 + 1056mn + 576m - 192n^3 - 1632n^2 - 4032n - 2592)x^2$$

$$+ (-12mn^2 - 24mn - 108m + 24n^2 + 144n + 216)x^4)/(6n^4 + 60n^3$$

$$+ 192n^2 + 228n + 90 + (2mn^3 + 12mn^2 + 22mn$$

$$+ 12m - n^3 - 7n^2 - 15n - 9)x^2). \quad (6.17)$$

The Padé solution obtained in (6.17) works for real values $n \geq 0$ and $m \geq 0$. For example, for fixed $n = 0$ and $m = 0$. The Padé $y(x)$ obtained in (6.17) is $y(x) = 1 - \frac{1}{2}x^2$, exact solution is $y(x) = 1 - \frac{1}{2}x^2$. for fixed $n = 0$ and $m = 1$. The Padé $y(x)$ obtained in (6.17) is $y(x) = \frac{30}{(30 + x^2)} - \frac{14x^2}{(30 + x^2)} + \frac{3x^4}{4(30 + x^2)}$, exact solution is $y(x) = \cos x$. Numerical solutions and of $y(x)$ for fixed $n = 0$ and $m = 1$ are shown in Table 1 and Figure 1.

Table 1. Numerical solution of $y(x)$ in (6.3) for fixed $n = 0$ and $m = 1$

t	Exact $y(x)$	p = Approx.$y(x)$	$\lvert y(x) - p \rvert$
0.0	1.00000	1.00000	0.000000
0.1	0.9950041653	0.9950041652	1×10^{-10}
0.2	0.9800665779	0.9800665779	0.000000
0.3	0.9553364906	0.9553364891	1.5×10^{-9}
0.4	0.9210609941	0.9210610079	1.38×10^{-8}
0.5	0.8775825622	0.8775826447	8.25×10^{-8}
0.6	0.8253356166	0.8253359685	3.519×10^{-7}
0.7	0.7648421951	0.7648433913	0.0000011962
0.8	0.6967067388	0.6967101829	0.0000034441
0.9	0.6216100638	0.6216187926	0.0000087288
1.0	0.5403025794	0.5403225807	0.0000200013

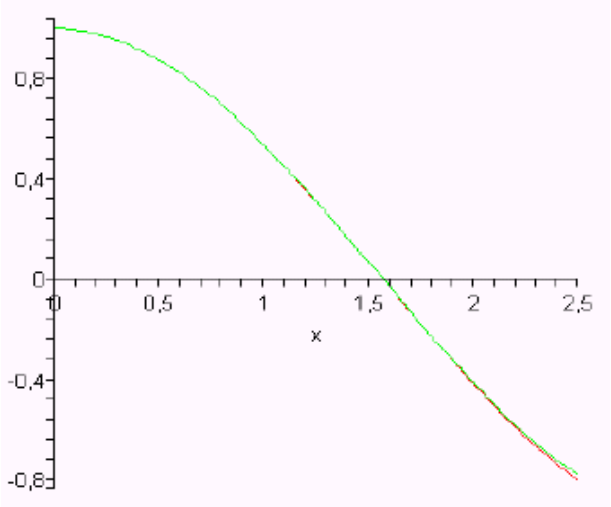


Figure 1. Values of $y(x)$, it is [4/3] Padé approximant for $n = 0$ and $m = 1$.

For fixed $n = 2$ and $m = 0$. The Padé $y(x)$ obtained in (6.17) is $y(x) = 1 - \frac{1}{6}x^2$, exact solution is $y(x) = 1 - \frac{1}{6}x^2$. For fixed $n = 2$ and $m = 1$. The Padé $y(x)$ obtained in (6.17) is $y(x) = \frac{42}{(42 + x^2)} - \frac{6x^2}{(42 + x^2)} + \frac{11x^4}{60(42 + x^2)}$, exact solution is $y(x) = \frac{\sin x}{x}$. Numerical solutions of $y(x)$ for fixed $n = 2$ and $m = 1$ are shown in Table 2. For fixed $n = 2$ and $m = 5$. The $y(x)$ Padé obtained in (6.17) is $y(x) = \frac{18}{(18 + 5x^2)} + \frac{2x^2}{(18 + 5x^2)} - \frac{x^4}{12(18 + 5x^2)}$, exact solution is $y(x) = \left(1 + \frac{1}{3}x^2\right)^{-1/2}$.

Solutions $y(x)$ for $n = 2$ and $m = 0$, 1, 2 and 3 are shown in Figure 2.

Table 2. Numerical solution of $y(x)$ in (6.3) for fixed $n = 2$ and $m = 1$

t	Exact $y(x)$	p = Approx.$y(x)$	$\lvert y(x) - p \rvert$
0.0	1.00000	1.00000	0.000000
0.1	0.9983341664	0.9983341662	2×10^{-10}
0.2	0.9933466539	0.9933466541	2×10^{-10}
0.3	0.9850673556	0.9850673558	2×10^{-10}
0.4	0.9735458557	0.9735458573	1.6×10^{-9}
0.5	0.9588510772	0.9588510847	7.5×10^{-9}
0.6	0.9410707892	0.9410708215	3.23×10^{-8}
0.7	0.9203109824	0.9203110928	1.1×10^{-7}
0.8	0.8966951162	0.8966954347	3.1×10^{-7}
0.9	0.8703632417	0.8703640503	8.0×10^{-7}
1.0	0.8414710096	0.8414728681	1.8×10^{-6}

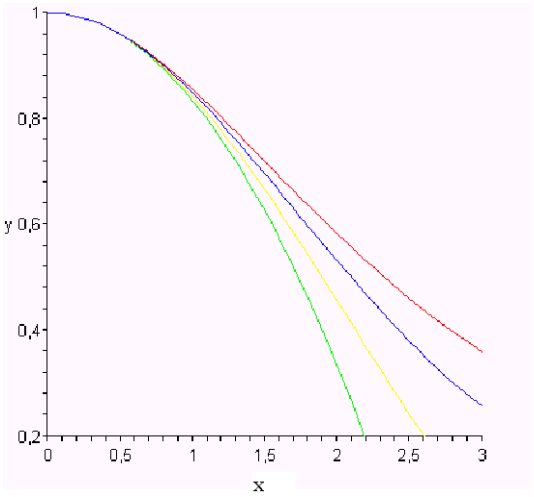


Figure 2. Values of $y(x)$, it is [4/3] Padé approximant for $n = 0$ and $m = 0,\ 1,\ 2$ and 3.

9. Conclusion

The method has proposed for solving Lane-Emden like equation with index-m in general fashion. Results show the advantages of the method. The presented solutions in this article are performed using MAPLE computer algebra systems [9].

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