Discover Applied Sciences

Research

On the structure of the higher dimensional Date-Jimbo-Kashiwara-Miwa model emerging in water waves

Kalim U. Tariq[1] · Jalil Manafian[2,3] · Somaye Malmir[4] · R. Nadir Tufail[1] · Onur Alp Ilhan[5] · Khaled H. Mahmoud[6]

Received: 14 October 2024 / Accepted: 12 May 2025
Published online: 31 May 2025
© The Author(s) 2025 OPEN

Abstract
In this paper, the (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa (DJKM) equation is investigated. The unified method, the modified Kudryashov scheme and the extended modified auxiliary equation mapping technique are employed to construct a variety of some new exact traveling wave solutions to the higher dimensional nonlinear model. Further, stability analysis and bifurcation analysis are employed to comprehend the dynamical system. Moreover, the physical structures of the observed solutions are demonstrated graphically. Via choosing appropriate amounts for the parameters contained in the obtained solutions, 3D plots, 2D contour plots and line plots are shown to graphically demonstrate the data. The reported results will be constructive in designing novel and better optical devices. The results of this work may also be useful in solving problems in marine engineering and nonlinear sciences.

Article Highlights

- Using the unified, modified Kudryashov and extended modified auxiliary equation mapping techniques
- Obtain various exact solutions
- The methods reduce the equation's complexity and significantly improves computational efficiency
- Stability analysis and bifurcation analysis

Keywords Date-jimbo-kashiwara-Miwa equation · The unified method · The modified kudryashov method · Traveling wave solution · The extended modified auxiliary equation mapping technique

1 Introduction

The research and discoveries in the fields of sciences and technology have always been aided by non-linear evolution equations (NLEEs). The NLEEs have been utilized to describe a wide range of physical and mathematical models, and their solutions have been vital in analyzing the properties and behaviors of these systems. The analytical solutions to NLEEs have been a hot topic of research in recent times. The unified method, the modified Kudryashov scheme and the

✉ Jalil Manafian, manafeian2@gmail.com | [1]Department of Mathematics, Mirpur University of Science and Technology, Mirpur, AJK 10250, Pakistan. [2]Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran. [3]Natural Sciences Faculty, Lankaran State University, 50, H. Aslanov str., Lankaran, Azerbaijan. [4]Department of Mathematics, Payame Noor University, 19395-4697, Tehran, Iran. [5]Faculty of Education, Department of Mathematics, Erciyes University, 38039 Melikgazi-Kayseri, Turkey. [6]Department of Physics, College of Khurma University College, Taif University, 11099, Taif 21944, Saudi Arabia.

extended modified auxiliary equation mapping technique based on Riccati equation form have been useful in finding soliton solutions, periodic solutions, and their interaction solutions. These solutions are acquired by using the series of expansion form of the NLEE. A dependent variable transformation is applied to the expansion form, resulting in the required solutions. It has been established that partial differential equations (PDEs) furnish a more effective explanation for physical phenomena than integer models, where these structures better describe dynamical systems. In mathematical physics, water waves, quantum mechanics, fluid mechanics, etc., nonlinear PDEs are encountered. It is difficult to discover solutions to complicated cases, especially nonlinear problems with higher dimensions containing new derivatives. To the best of our knowledge, there is no arbitrary idea for handling some nonlinear issues with larger dimensions. Analytical solutions to nonlinear partial differential equations (NPDEs) is a growing area of research interest in the engineering sciences, mathematical physics, and other technological domains. The dynamical systems have been extensively used in real-world applications, especially biology, ecology, physics, economics, and sociology. They are an important tool for the analysis of the complex dynamics that a system displays, including instability, and water wave systems. The exact or numerical solutions to nonlinear equations, which describe phenomena across different arenas, have become an significant area of research, leading to the development of plentiful techniques by scientists working in the topic. The methods that have been developed include the multi-soliton rational solutions, the exp-function method, the Gaussian solitary waves method, the $(G'/G, 1/G)$-expansion method and the generalized exponential rational function approach [1–9]. In later decades, the nonlinear advancement conditions (NLEEs) have been considered and utilized to recreate different occasions in physics, chemistry, science and indeed the social sciences. Since the 1950 s, the notion of soliton has been presented into the think about of nonlinear marvels, making the ponder of the solutions of NLPDEs a hot spot in nonlinear sciences. Be that as it may, due to the complexity of NLPDEs, the comes about of scientific inquire about at display have not given a all around compelling way to find the correct solution. With the rise of a assortment of solution ways, not as it were the past troublesome to unravel the condition has been illuminated, but moreover modern, vital physical meaning of the solution has been discovered and connected in hone. Luckily, through the persistent endeavors of mathematicians and physicists, we understood that soliton hypothesis contains a solution of compelling strategies to develop precise arrangements, such as different exp-function strategy, Bäcklund change, Darboux change, Hirota bilinear strategy, Lie bunch strategy, etc. In arrange to create different correct solutions of nonlinear PDEs, a modern strategy named bilinear neural method is suggested and the comparing tensor equation is built by Zhang et al. Due to the solid nonlinear characteristics of neural organize models, the application of neural arrange models in tackling NLEEs has pulled in the consideration of analysts [10–20].

Analysts from around the world have found a range of numerical and expository strategies to study the behaviour of nonlinear models, counting the Hirota bilinear method [21, 22], the hyperbolic ansatz method [23, 24], the improved F-expansion method [25, 26], the modified auxiliary equation method [27–29], the deep convolutional neural method [30], the optimal connected deep neural method [31], the deep neural learning mathematical modeling [32], the extended tanh function method [33, 34], the matrix factorizations [35], the extended trial function technique [36–38], the fuzzy logic [39], the extended simple equation method [40, 41],the $\exp(-\phi(\xi))$ expansion scheme [42, 43] and the Adomian decomposition method [44, 45].

The DJKM model describes the weakly nonlinear restoring forces and long water waves with frequency dispersion. This demonstrate has been examined employing a variety of capable techniques within the writing, counting, the improved Bernoulli sub-equation method [46], the two-variable $(\frac{G'}{G}, \frac{1}{G})$-expansion technique [47], the extended direct algebraic method [48], the Lie symmetry method [49], the modified *F*-expansion technique [50] and the finite difference method [51].

The influence of fractional derivatives on solutions of different physical forms within a single mathematical structure by investigating the Kadomtsev-Petviashvili-Benjamin-Bona-Mahony equation has been made [52]. Al-Shara et al used the necessary conditions for the existence of optical solitary solutions within a novel two-mode extension derived from the unstable nonlinear Schrödinger equation (NLSE) [53]. The rational sine-cosine functions and the Kudryashov-expansion method were used to study new optical solitons to the generalized nonlinear Schrödinger equation [54]. A generalized fifth-order equation with third-order time-dispersion term was investigated by implementing three functional-methods [55]. New exact solutions of the coupled nonlinear Schrödinger (NLS) system using the generalized G-expansion method and the modulation instability analysis were extracted [56]. A non-autonomous (3+1) dimensional coupled NLS equation with variable coefficients in optical fiber communication was analyzed via the Hirota bilinear technique [57]. The double periodic soliton, breather wave and the multiple rogue wave solutions to a generalized breaking soliton system were investigated by the Hirota bilinear frame [58]. The (1+1)- and (2+1)-dimensional Chiral NLS equations were studied with Modified extended tanh-function method

[59]. The numerous rogue wave solutions using the Hirota bilinear hypothesis were obtained to the generalized Hirota-Satsuma-Ito condition [60]. The potential KP type coupled system with variable coefficients was studied to investigate the soliton interactions in a two-dimensional space with time [61]. The solitary wave dynamics of the thin-film ferroelectric material equation were achieved utilizing the standard $\tan(\phi/2)$-expansion scheme and generalized G-expansion technique [62]. The spatial symmetric nonlinear dispersive wave model in (2+1)-dimensions was investigated using the Hirota bilinear form [63]. Soliton, bright-dark soliton and periodic were studied with usage of the improved $\tan(\phi/2)$-expansion technique and Paul-Painlevé analysis [64]. The exact soliton solutions to the quartic Rosenau-Kawahara-Regularized-Long-Wave fluid model with fractional derivative were obtained using the modified extended tanh function method and the improved (G'/G)-expansion technique [65]. An efficient numerical method based on the collocation method was applied to KdV-Burger-Fisher equation which is one of the dispersion-dissipation-reaction model [66]. A reductive perturbation method was used to generate the trapped Zakharov-Kuznetsov Burgers' equation where have investigated the propagation of nonlinear electrostatic shock waves in an anisotropically pressured magnetized plasma containing positive and negative ions, trapped electrons, and positrons [67]. The weighted residual technique was used to produce a solitary type progressive wave solution for very small values of the Burgers term [68].

The X-shape soliton and Y-shape soliton solutions were studied via the Hirota bilinear approach for the (2+1)-dimensional Boussinesq equation [69]. Wang solved the generalized (3+1)-dimensional B-type KP equation using the Cole-Hopf transform and Hirota bilinear equation [70]. Variational principle, Hamiltonian, bifurcation analysis, chaotic behaviors and the diverse solitary wave solutions were investigated to solve the simplified modified Camassa-Holm equation [71]. The new extended (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation was studied using the Cole-Hopf transformation and the bilinear form [72].

In our work, the unified method [74], the modified Kudryashov technique [75] and the extended modified auxiliary equation mapping (AEM) technique [76] are utilized to study the DJKM model [77] read as

$$\begin{aligned} &g_{xxxxy} + 4g_{xxy}g_x + 2g_{xxx}g_y + 6g_{xy}g_{xx} \\ &\quad - \alpha g_{yyy} - 2\beta g_{xxt} = 0, \end{aligned} \tag{1}$$

where $g(x, y, t)$ is the wave amplitude function and t is time. The application of rational form solutions involving sine-cosine and sinh-cosh functions have been examined to solve the Date-Jimbo-Kashiwara-Miwa equation in [78].

The (2+1)-dimensional Sawada-Kotera-Kadomtsev Petviashvili equation was investigated using the Hirota bilinear method to obtain the lump wave solutions [79]. The bifurcation and chaotic analysis of the time-fractional Benjamin Ono equation in the conformable sense for shallow-water waves were studied in [80]. Some different localized wave solutions including the bright and dark rogue waves solutions to the (3+1)-dimensional Kudryashov-Sinelshchikov equation for the liquid with gas bubbles were explored [81]. Using the traveling wave transformation and semi-inverse method based on the Hamiltonian was extracted to integrable Kadomtsev-Petviashvili equation [82].

The essential objective of this inquire about is to offer the higher dimensional Date-Jimbo-Kashiwara-Miwa model emerging in water waves a noteworthy number of reliable expository closed-form solutions. In light of the findings, it may be said that the study's motivation that is, that the data are novel, fascinating, and potentially very important in helping us understand how the methods behaves. Furthermore, our work's novelty is justified because, upon comparing our results to those previously obtained through various approaches, it becomes evident that while some solution types have already been documented in the literature, the majority are new. The current study has systematically derived several novel solitary wave solutions for the Date-Jimbo-Kashiwara-Miwa model, employing the unified method, the modified Kudryashov scheme and the extended modified auxiliary equation mapping technique.

The article is structured in the following:

In section 2, the algorithms of three methods are mentioned. In section 3, applications of the mentioned techniques to solve Eq. (1) are stated. Section 4 presents the discussions of obtained solutions and implementation of the graphical representations. Sections 5 and 6 are used to investigate the stability analysis and bifurcation analysis. Finally, a concluding summary of the research is presented in section 7.

2 Description of techniques

Firstly consider the NLPDE as

$$F(g, g_t, g_x, g_y, g_{xt}, g_{xx}, g_{yt}, g_{yy}, \cdots) = 0, \tag{2}$$

here F is the function of g and its partial derivatives.

To obtain the exact traveling wave solution, we seek the transformation of the form

$$g(x, y, t) = \Omega(\xi), \qquad \xi = ax + by - ct, \tag{3}$$

here a, b are constants and c is the speed of the wave. The transformation given by (3) changes (2) into ODE as

$$H(\Omega, \Omega', \Omega'', \Omega''', \cdots) = 0, \tag{4}$$

here H is the function of Ω and its derivatives.

2.1 The unified technique

The taking after are the major steps within the bound together strategy:

Step 1. Let the solution to (4) can be stated as

$$\phi(\xi) = a_0 + \sum_{i=1}^{n} \left(a_i \theta^i(\xi) + b_i \theta^{-i}(\xi)\right), \tag{5}$$

where a_i, b_i are obscure constants, n could be a positive numbers and $\theta(\xi)$ fulfils

$$\frac{\mathrm{d}\theta}{\mathrm{d}\xi} = \theta(\xi)^2 + \lambda, \tag{6}$$

where λ is a real parameter. So, we obtain the following exact solutions.

2.2 Family 1

If $\lambda < 0$, then

$$\phi(\xi) = \frac{\pm\sqrt{-\left(\zeta^2 + \eta^2\right)\lambda} - \zeta\sqrt{-\lambda}\cosh\left(2\sqrt{-\lambda}\left(\xi + \xi_0\right)\right)}{\zeta\sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) + \eta}, \tag{7}$$

$$\phi(\xi) = \pm\sqrt{-\lambda} + \frac{\mp 2\zeta\sqrt{-\lambda}}{\zeta + \left(\cosh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) \mp \sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right)\right)}. \tag{8}$$

2.3 Family 2

If $\lambda > 0$, then

$$\phi(\xi) = \frac{\pm\sqrt{\left(\zeta^2 + \eta^2\right)\lambda} - \zeta\sqrt{\lambda}\cos\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right)}{\zeta\sin\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right) + \eta}, \tag{9}$$

$$\phi(\xi) = \pm\sqrt{\lambda}i + \frac{\mp 2\zeta i\sqrt{\lambda}}{\zeta + \left(\cos\left(2(\xi+\xi_0)\sqrt{\lambda}\right) \mp i\sin\left(2(\xi+\xi_0)\sqrt{\lambda}\right)\right)}, \lambda < 0. \tag{10}$$

2.4 Family 3

If $\lambda = 0$, then

$$\phi(\xi) = -\frac{1}{\xi + \xi_0}, \tag{11}$$

where ζ, η and ξ_0 are real arbitrary parameters.

Step 2. We can find n in (5) by balancing the highest order derivatives and the nonlinear terms in (4).
Step 3. We get a framework of equations by putting (5) and (6) into (4) and equating the coefficients of θ^j to zero.
Step 4. By understanding the framework in step 3, we accomplish the precise arrangements for (2).

2.5 The modified kudryashov method

Step 1. Suppose the solution of (4) in the form:

$$\phi(\xi) = \sum_{j=0}^{n} k_j \theta^j(\xi), \tag{12}$$

where k_j are unknown constants, and

$$\theta(\xi) = \frac{1}{1 + dk^{\xi}}, \tag{13}$$

satisfies

$$\theta'(\xi) = \ln(k)\left(\theta(\xi)^2 - \theta(\xi)\right),\ k \neq 0, 1. \tag{14}$$

Step 2. We can calculate n in (12) by balancing higher order derivatives and nonlinear terms in (4).
Step 3. Putting (12) and (13) into (4) and equating the coefficients of θ^j to zero, we attain a set of equations.
Step 4. Then we use the Mathematica 18 package to solve this system of equations. we find exact solutions for (2) by putting the attained values in (12).

2.6 The extended modified AEM technique

Step 1. Let the solution to (4) is given by

$$\phi(\xi) = \sum_{k=0}^{n} a_k \theta^k(\xi) + \sum_{k=-1}^{-n} b_{-k} \theta^k(\xi) + \sum_{k=2}^{n} c_k \theta^{k-2}(\xi)\theta'(\xi) + \sum_{k=1}^{n} d_k \frac{\theta'(\xi)}{\theta(\xi)}, \tag{15}$$

where a_k, b_k, c_k, d_k are the unknown constants and $\theta(\xi)$ satisfies

$$\theta'(\xi)^2 = \delta_1\theta^2(\xi) + \delta_2\theta^3(\xi) + \delta_3\theta^4(\xi), \tag{16}$$

where $\delta_i (i = 1, 2, 3)$ are constants.

Step 2. Equating the highest order derivative and nonlinear terms in (2), we can obtain n in (15)

Step 3. Putting (3), (15) and (16) in (2) and equating the coefficients of $\theta'^j\theta^k$, a set of equations can be obtained.

Step 4. Appending all the obtained amounts in (15), we find exact solutions of (2).

3 Analytical solutions

We use the transformation (3) in (1) to get an ODE as

$$\begin{aligned} &6a^3b\Omega''(\xi)^2 - b^3\alpha\Omega^{(3)}(\xi) + 2a^2\beta c\Omega^{(3)}(\xi) \\ &\quad + 6a^3b\Omega^{(3)}(\xi)\Omega'(\xi) + a^4b\Omega^{(5)}(\xi) = 0. \end{aligned} \tag{17}$$

After simplification, we get

$$\begin{aligned} &- b^3\alpha\Omega''(\xi) + 2a^2c\beta\Omega''(\xi) + 6a^3b\Omega'(\xi)\Omega''(\xi) \\ &\quad + a^4b\Omega^{(4)}(\xi) = 0. \end{aligned} \tag{18}$$

Balancing $\Omega^{(4)}(\xi)$ with $(\Omega'(\xi)\Omega''(\xi))$ in (18) we obtain $n + 4 = 2n + 3$, so that $n = 1$.

3.1 Applications of the unified method

Fom (5), we can write as

$$g(\xi) = a_0 + a_1\theta(\xi) + b_1\theta^{-1}(\xi), \tag{19}$$

now inserting (19) along with (6) into (18) and collecting the coefficients of $\theta^i(\xi)$, we obtain the system of equations:

$$\begin{aligned} &12a_1a^4b + 6a_1^2a^3b = 0, \\ &12a^4bb_1\lambda^4 - 6a^3bb_1^2\lambda^3 = 0, \\ &b_1\lambda\left(8a^4b\lambda + 6a_1a^3b\lambda + 2a^2\beta c - \alpha b^3\right) - 6a^3bb_1^2\lambda = 0, \\ &20a_1a^4b\lambda + 12a_1^2a^3b\lambda - 6a_1a^3bb_1 + 2a_1a^2\beta c - \alpha a_1b^3 = 0, \\ &8a_1a^4b\lambda^2 + 6a_1^2a^3b\lambda^2 - 6a_1a^3bb_1\lambda + 2a_1a^2\beta c\lambda - \alpha a_1b^3\lambda = 0, \\ &12a^4bb_1\lambda^3 - 12a^3bb_1^2\lambda^2 + b_1\lambda^2\left(8a^4b\lambda + 6a_1a^3b\lambda + 2a^2\beta c - \alpha b^3\right) = 0, \end{aligned}$$

we find the different results of this system:

3.2 Set 1

$$a_1 = -2a,\ b_1 = 2a\lambda,\ a = a,\ b = b,\quad c = -\frac{-16a^4b\lambda - \alpha b^3}{2a^2\beta}. \tag{20}$$

3.3 Set 2

$$a_1 = -2a,\ b_1 = 0,\ a = a,\ b = b,\quad c = -\frac{-4a^4b\lambda - \alpha b^3}{2a^2\beta}. \tag{21}$$

3.4 Set 3

$$a_1 = 0,\ b_1 = 2a\lambda,\ a = a,\ b = b,\quad c = -\frac{-4a^4b\lambda - \alpha b^3}{2a^2\beta}. \tag{22}$$

For Set 1, we develop the traveling wave arrangements of (1) as takes after,

If $\lambda < 0$, then

$$g_1^1(x,y,t) = a_0 - \frac{(2a)\left(\sqrt{-\lambda\left(\zeta^2+\eta^2\right)} - \zeta\sqrt{-\lambda}\cosh\left(2\sqrt{-\lambda}\left(\xi+\xi_0\right)\right)\right)}{\zeta\sinh\left(2\sqrt{-\lambda}\left(\xi+\xi_0\right)\right)+\eta} + 2a\lambda\left(\frac{\sqrt{-\lambda\left(\zeta^2+\eta^2\right)} - \zeta\sqrt{-\lambda}\cosh\left(2\sqrt{-\lambda}\left(\xi+\xi_0\right)\right)}{\zeta\sinh\left(2\sqrt{-\lambda}\left(\xi+\xi_0\right)\right)+\eta}\right)^{-1},$$

$$g_2^1(x,y,t) = a_0 - 2a\left(\sqrt{-\lambda} + \frac{-2\zeta\sqrt{-\lambda}}{\zeta+\left(\cosh\left(2\left(\xi+\xi_0\right)\sqrt{-\lambda}\right)-\sinh\left(2\left(\xi+\xi_0\right)\sqrt{-\lambda}\right)\right)}\right) + 2a\lambda\left(\sqrt{-\lambda} + \frac{-2\zeta\sqrt{-\lambda}}{\zeta+\left(\cosh\left(2\left(\xi+\xi_0\right)\sqrt{-\lambda}\right)-\sinh\left(2\left(\xi+\xi_0\right)\sqrt{-\lambda}\right)\right)}\right)^{-1},$$

where $\xi = ax + by - \frac{\left(-16a^4b\lambda-\alpha b^3\right)t}{2a^2\beta}$.

If $\lambda > 0$, then

$$g_3^1(x,y,t) = a_0 - \frac{(2a)\left(\sqrt{\lambda\left(\zeta^2-\eta^2\right)} - \zeta\sqrt{\lambda}\cos\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)\right)}{\zeta\sin\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)+\eta} + 2a\lambda\left(\frac{\sqrt{\lambda\left(\zeta^2-\eta^2\right)} - \zeta\sqrt{\lambda}\cos\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)}{\zeta\sin\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)+\eta}\right)^{-1},$$

$$g_4^1(x,y,t) = a_0 - 2a\left(i\sqrt{\lambda} + \frac{-2\zeta i\sqrt{\lambda}}{\zeta+\left(\cos\left(2\sqrt{\lambda}\left(\xi+\xi_0\right)\sqrt{\lambda}\right)-i\sin\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)\right)}\right) + 2a\lambda\left(i\sqrt{\lambda} + \frac{-2\zeta i\sqrt{\lambda}}{\zeta+\left(\cos\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)-i\sin\left(2\left(\xi+\xi_0\right)\sqrt{\lambda}\right)\right)}\right)^{-1},$$

where $\xi = ax + by - \frac{\left(-16a^4b\lambda-\alpha b^3\right)t}{2a^2\beta}$.

If $\lambda = 0$, then

$$g_5^1(x,y,t) = a_0 + \frac{2a}{\xi + \xi_0},$$

where $\xi = ax + by - \frac{(-16a^4b\lambda - \alpha b^3)t}{2a^2\beta}$.

For Set 2, we develop the traveling wave arrangements of (1) as takes after,

If $\lambda < 0$, then

$$g_6^1(x,y,t) = a_0 - 2a \left(\frac{\sqrt{-\lambda\left(\zeta^2 + \eta^2\right)} - \zeta\sqrt{-\lambda}\cosh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right)}{\zeta\sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) + \eta} \right),$$

$$g_7^1(x,y,t) = a_0 - 2a \left(\sqrt{-\lambda} + \frac{-2\zeta\sqrt{-\lambda}}{\zeta + \left(\cosh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) - \sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right)\right)} \right),$$

where $\xi = ax + by - \frac{-4a^4b\lambda - \alpha b^3 t}{2a^2\beta}$.

If $\lambda > 0$, then

$$g_8^1(x,y,t) = a_0 - 2a \left(\frac{\sqrt{\lambda\left(\zeta^2 - \eta^2\right)} - \zeta\sqrt{\lambda}\cos\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right)}{\zeta\sin\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right) + \eta} \right),$$

$$g_9^1(x,y,t) = a_0 - 2a \left(i\sqrt{\lambda} + \frac{-2\zeta i\sqrt{\lambda}}{\zeta + \left(\cos\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right) - i\sin\left(2\left(\xi + \xi_0\right)\sqrt{\lambda}\right)\right)} \right),$$

where $\xi = ax + by - \frac{-4a^4b\lambda - \alpha b^3 t}{2a^2\beta}$.

If $\lambda = 0$, then

$$g_{10}^1(x,y,t) = a_0 + \frac{2a}{\xi + \xi_0},$$

where $\xi = ax + by - \frac{-4a^4b\lambda - \alpha b^3 t}{2a^2\beta}$.

For Set 3, we develop the traveling wave arrangements of (1) as takes after,

If $\lambda < 0$, then

$$g_{11}^1(x,y,t) = a_0 + 2a\lambda \left(\frac{\sqrt{-\lambda\left(\zeta^2 + \eta^2\right)} - \zeta\sqrt{-\lambda}\cosh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right)}{\zeta\sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) + \eta} \right)^{-1},$$

$$g_{12}^1(x,y,t) = a_0 + 2a\lambda \left(\sqrt{-\lambda} + \frac{-2\zeta\sqrt{-\lambda}}{\zeta + \left(\cosh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right) - \sinh\left(2\left(\xi + \xi_0\right)\sqrt{-\lambda}\right)\right)} \right)^{-1},$$

where $\xi = ax + by - \frac{-4a^4b\lambda - \alpha b^3 t}{2a^2\beta}$.

If $\lambda > 0$, then

$$g_{13}^{1}(x,y,t) = a_0 + 2a \left(\lambda \left(\frac{\sqrt{\lambda(\zeta^2 - \eta^2)} - \zeta\sqrt{\lambda}\cos\left(2(\xi+\xi_0)\sqrt{\lambda}\right)}{\zeta\sin\left(2(\xi+\xi_0)\sqrt{\lambda}\right) + \eta} \right)^{-1},$$

$$g_{14}^{1}(x,y,t) = a_0 + 2a \left(\lambda \left(i\sqrt{\lambda} + \frac{-2\zeta i\sqrt{\lambda}}{\zeta + \left(\cos\left(2(\xi+\xi_0)\sqrt{\lambda}\right) - i\sin\left(2(\xi+\xi_0)\sqrt{\lambda}\right)\right)} \right)^{-1},$$

where $\xi = ax + by - \frac{-4a^4b\lambda - \alpha b^3 t}{2a^2\beta}$.

3.5 Applications of the modified Kudryashov method

From (12), we assume the solution for (18) which can be given in the form:

$$g(\xi) = k_0 + k_1\theta(\xi). \tag{23}$$

Substituting (23) along with (14) into (18) and equating the coefficients of $\theta^i(\xi)$ to zero, we get the system of equations:

$$\begin{aligned}
&\theta^5 : 24a^4bk_1(\ln(k))^4 + 12a^3bk_1^2(\ln(k))^3 = 0,\\
&\theta^4 : -60a^4k_1b(\ln(k))^4 - 30a^3k_1^2b(\ln(k))^3 = 0,\\
&\theta^3 : 50a^4bk_1(\ln(k))^4 +\\
&24a^3bk_1^2(\ln(k))^3 + 4a^2\beta ck_1(\ln(k))^2 - 2\alpha b^3k_1(\ln(k))^2 = 0,\\
&\theta^2 : -15a^4bk_1(\ln(k))^4 - 6a^3bk_1^2(\ln(k))^3 - 6a^2\beta ck_1(\ln(k))^2\\
&+ 3\alpha b^3k_1(\ln(k))^2 = 0,\\
&\theta : a^4bk_1(\ln(k))^4 + 2a^2\beta ck_1(\ln(k))^2 - \alpha b^3k_1(\ln(k))^2 = 0.
\end{aligned}$$

We get the taking after result by tackling this framework:

$$k_1 = -2a\ln(k),\quad a = a,\quad b = b,\quad c = -\frac{a^4b(\ln(k))^2 - \alpha b^3}{2a^2\beta}. \tag{24}$$

We get the following exact solution using these parameters

$$g_1^2(x,y,t) = k_0 - \frac{2a\ln(k)}{1 + dk^{\xi}}, \tag{25}$$

where $\xi = ax + by - \frac{a^4b(\ln(k))^2 - \alpha b^3 t}{2a^2\beta}$ and k_0, d are real arbitrary constants.

3.6 Applications of the extended modified AEM technique

Similarly, the solution (15) can be expressed as

$$q(\xi) = a_0 + a_1\theta(\xi) + \frac{b_1}{\theta(\xi)} + d_1\frac{\theta'(\xi)}{\theta(\xi)}. \tag{26}$$

Putting (26) and (16) into (18) and collecting the coefficients of $\theta'^{j}\theta^{(k)}$ to zero, we get the system of equations:

$$
\begin{aligned}
&24a^4bd_1 - 12a^3bd_1^2 = 0,\\
&24a^4bb_1 - 24a^3bb_1d_1 = 0,\\
&12a_1a^4b\delta_3^2 + 24a_1a^3bd_1\delta_3^2 = 0,\\
&15a_1a^4b\delta_2\delta_3 + 36a_1a^3bd_1\delta_2\delta_3 = 0,\\
&24a^3bd_1^2\delta_3 - 48a^4bd_1\delta_3 = 0,\\
&27a^3bd_1^2\delta_2 - 45a^4bd_1\delta_2 = 0,\\
&12a^4a_1b\delta_3 - 12a^3a_1bd_1\delta_3 = 0,\\
&48a^4bd_1\delta_3^2 + 12a_1^2a^3b\delta_3 = 0,\\
&3a^4a_1b\delta_2 - 18a^3a_1bd_1\delta_2 = 0,\\
&36a^3bb_1d_1\delta_2 - 33a^4bb_1\delta_2 = 0,\\
&16a^4bb_1\delta_1\delta_3 - 24a^3bb_1d_1\delta_1\delta_3 + 4a^2\beta b_1c\delta_3 = 0,\\
&5a^4bb_1\delta_1^2 - 6a^3bb_1d_1\delta_1^2 - 2a^2\beta b_1c\delta_1 + \alpha b^3b_1\delta_1 = 0,\\
&63a^4bd_1\delta_2\delta_3 - 18a^3bd_1^2\delta_2\delta_3 + 9a_1^2a^3b\delta_2 = 0,\\
&-36a^4bb_1\delta_3 - 18a_1a^3bd_1\delta_1 + 36a^3bb_1d_1\delta_3 = 0,\\
&-28a^4bb_1\delta_1 + 30a^3bb_1d_1\delta_1 + 4a^2\beta b_1c - 2\alpha b^3b_1 = 0,\\
&-40a^4bd_1\delta_1 + 24a^3bd_1^2\delta_1 + 12a_1a^3bb_1 + 4a^2\beta cd_1 - 2\alpha b^3d_1 = 0,\\
&\frac{27}{2}a^4bb_1\delta_1\delta_2 - 18a^3bb_1d_1\delta_1\delta_2 - 3a^2\beta b_1c\delta_2\\
&+ \frac{3}{2}\alpha b^3b_1\delta_2 = 0,\\
&\frac{75}{2}a^4bd_1\delta_1\delta_2 - 27a^3bd_1^2\delta_1\delta_2 - 18a_1a^3bb_1\delta_2 - 3a^2\beta cd_1\delta_2\\
&+ \frac{3}{2}\alpha b^3d_1\delta_2 = 0,\\
&16a^4bd_1\delta_1^2 - 12a^3bd_1^2\delta_1^2 - 12a_1a^3bb_1\delta_1\\
&+ 12a^3bb_1^2\delta_3 - 4a^2\beta cd_1\delta_1 + 2\alpha b^3d_1\delta_1 = 0,\\
&\frac{45}{2}a^4bd_1\delta_2^2 + 48a^4bd_1\delta_1\delta_3\\
&- \frac{27}{2}a^3bd_1^2\delta_2^2 - 24a^3bd_1^2\delta_1\delta_3 + 6a_1^2a^3b\delta_1 - 24a_1a^3bb_1\delta_3 = 0,\\
&\frac{9}{2}a_1a^4b\delta_1\delta_2 + 21a^4bb_1\delta_2\delta_3 + 18a_1a^3bd_1\delta_1\delta_2\\
&- 36a^3bb_1d_1\delta_2\delta_3 + 3a_1a^2\beta c\delta_2 - \frac{3}{2}\alpha a_1b^3\delta_2 = 0,\\
&\frac{9}{2}a_1a^4b\delta_2^2 + 12a^4bb_1\delta_3^2 + 8a_1a^4b\delta_1\delta_3 + \frac{27}{2}a_1a^3bd_1\delta_2^2\\
&- 24a^3bb_1d_1\delta_3^2 + 24a_1a^3bd_1\delta_1\delta_3 + 4a_1a^2\beta c\delta_3 - 2\alpha a_1b^3\delta_3 = 0,\\
&a_1a^4b\delta_1^2 + 9a^4bb_1\delta_2^2 + 16a^4bb_1\delta_1\delta_3 + 6a_1a^3bd_1\delta_1^2\\
&- \frac{27}{2}a^3bb_1d_1\delta_2^2 - 24a^3bb_1d_1\delta_1\delta_3 - 4a^2\beta b_1c\delta_3\\
&+ 2a_1a^2\beta c\delta_1 - \alpha a_1b^3\delta_1 + 2\alpha b^3b_1\delta_3 = 0,
\end{aligned}
$$

we find the different results of this system:

Family I

$$
\begin{aligned}
&a_1 = 0,\ b_1 = 0,\ d_1 = 2a,\ \ a = a,\ \ \ b = b,\ \ \ c\\
&\quad = \frac{\alpha b^3 - 4a^4b\delta_1}{2a^2\beta},\ \delta_2 = 0,\ \delta_3 = 0,
\end{aligned}
\tag{27}
$$

so that the solution to (1) is given by

$$g_1^3(x,y,t) = a_0 + \frac{2a\sqrt{\delta_1\theta(\xi)^2}}{\theta(\xi)}, \tag{28}$$

where

$$\theta(\xi) = -\frac{4\delta_1 e^{\sqrt{\delta_1}\xi}}{\delta_2^2\left(-e^{2\sqrt{\delta_1}\xi}\right) + 2\delta_2 e^{\sqrt{\delta_1}\xi} + 4\delta_1\delta_3 e^{2\sqrt{\delta_1}\xi} - 1}, \tag{29}$$

where $\xi = ax + by - \frac{\alpha b^3 - 4a^4 b\delta_1}{2a^2\beta}t$.

Family II

$$\begin{aligned} a_1 = 0,\ b_1 = 0,\ d_1 = 2a,\ \ a = a,\ \ b = b,\ \ c \\ = \frac{\alpha b^3 - 4a^4 b\delta_1}{2a^2\beta},\ \delta_2 = 0,\ \delta_3 = 0, \end{aligned} \tag{30}$$

so that the solution to (1) can be expressed as

$$g_2^3(x,y,t) = a_0 + \frac{2a\sqrt{\delta_1\theta(\xi)^2}}{\theta(\xi)}, \tag{31}$$

where

$$\theta(\xi) = \frac{4\delta_1 e^{\sqrt{\delta_1}\xi}}{-2\delta_2 e^{\sqrt{\delta_1}\xi} + e^{2\sqrt{\delta_1}\xi} + \delta_2^2 - 4\delta_1\delta_3}, \tag{32}$$

where $\xi = ax + by - \frac{\alpha b^3 - 4a^4 b\delta_1}{2a^2\beta}t$.

4 Discussion and results

The graphical representations of the DJKM model are shown in this section. Using the methods discussed, several forms of travelling waves solutions of (1) can be produced. To superior get it the conduct of arrangements, 3D, form and 2D charts of correct arrangements with different parameter values are appeared.

5 Stability analysis

Here, we have investigated the stability of the DJKM equation using the linear stability analysis method. This technique involves introducing a perturbed wave solution (PWS) to the equilibrium state in order to analyze the model's behavior under perturbation. Take the following function as:

$$\begin{aligned} g(x,y,t) &= (A_0 + u(x,y,t))\,h(t), \\ h(t) &= \exp(A_0\,B_0\,t), \end{aligned} \tag{33}$$

where A_0 is the steady state solution, B_0 is a constant and $g(x,y,t)$ is the PWS presented in model (1). Substituting PWS in the governing equation (1) and linearizing, we obtain

$$\begin{aligned} &2B_0A_0\beta\left(\tfrac{\partial^2}{\partial x^2}u(x,y,t)\right) + 2\beta\left(\tfrac{\partial^3}{\partial t\partial x^2}u(x,y,t)\right) + \\ &\left(\tfrac{\partial^3}{\partial y^3}u(x,y,t)\right)\alpha - \tfrac{\partial^5}{\partial x^4\partial y}u(x,y,t) = 0. \end{aligned} \tag{34}$$

Let us presume that the linear differential equation (34) holds a solution of the form:

$$u(x,y,t) = \rho_1 e^{\chi t+\sigma y+\tau x}, \tag{35}$$

where χ is the frequency of perturbation and σ, τ normalized wave numbers. Then, we obtain the following dispersion relation as:

$$\chi(\sigma,\tau) = -\frac{2B_0A_0\beta\,\tau^2 - \tau^4\sigma + \sigma^3\alpha}{2\beta\,\tau^2}. \tag{36}$$

The above equation of dispersion relation exhibits that the stability of steady-state depends on the modulation of self-phase, stimulated Raman scattering, frequency and dispersion of group velocity. We investigate the stability in Figures (8, 910).

6 Equilibrium state and bifurcation analysis

Bifurcation analysis in dynamic methods investigates a phenomenon where small variations in system parameters result in qualitative variations in the system's behavior. The bifurcation hypothesis explains these unexpected events as well as predicts how structures will behave at different stages of their evolution.

$$\begin{aligned} &-b^3\alpha\Omega''(\xi) + 2a^2c\beta\Omega''(\xi) + 6a^3b\Omega'(\xi)\Omega''(\xi) \\ &\quad + a^4b\Omega^{(4)}(\xi) = 0. \end{aligned} \tag{37}$$

After once integrating respect to ξ, we get

$$a^4b\Omega^{(3)}(\xi) + (2a^2c\beta - b^3\alpha)\Omega'(\xi) + 3a^3b\Omega'(\xi)^2 = 0. \tag{38}$$

or

$$\Omega^{(3)}(\xi) + \delta_1\Omega'(\xi) + \delta_2\Omega'(\xi)^2 = 0. \tag{39}$$

where $\delta_1 = \frac{(2a^2c\beta - b^3\alpha)}{a^4b}$ and $\delta_2 = \frac{3}{a}$. Let $\frac{d\Omega}{d\xi} = X$, the planner dynamics of the governing problem for equation Eq. (39) can be stated as below

$$\begin{cases} \frac{dX}{d\xi} = Y = f(X,Y), \\ \frac{dY}{d\xi} = -\delta_1X - \delta_2X^2 = g(X,Y), \end{cases} \tag{40}$$

with assessed Hamiltonian function which is identified as solution of the dynamical scheme, is

$$H(X,Y) = \frac{1}{2}Y^2 + \frac{1}{2}\delta_1X^2 + \frac{1}{3}\delta_2X^3 = h, \tag{41}$$

and h characterizes a continuous variable, also called the Hamiltonian constants or energy levels. Moreover, $\frac{1}{2}Y^2$ denotes the kinetic energy, and $\frac{1}{2}\delta_1X^2 + \frac{1}{3}\delta_2X^3$ represents the potential energy for the Hamiltonian Eq. (41). Critical points in the system, where no variations occur, are identified by solving the condition $\nabla H(X,Y) = 0$. for the phase portrait trajectories of the system (40), fixed points or equilibria of Eq. (40) are obtained by utilizing the following results:

$$\begin{cases} \frac{dX}{d\xi} = 0, \\ \frac{dY}{d\xi} = 0, \end{cases} \tag{42}$$

which becomes:

$$Y = 0, \ \ and \ \ -\delta_1X - \delta_2X^2 = 0. \tag{43}$$

Then, we have

$$Y = 0, \ \ and \ X = 0, -\frac{\delta_1}{\delta_2} = -\frac{2a^2c\beta - b^3\alpha}{3a^3\,b}. \tag{44}$$

Hence, we have two equilibrium points $O(X_0, 0)$ and $E(X_1, 0)$ with the values:

$$X_0 = 0, \ \ X_1 = -\frac{2a^2c\beta - b^3\alpha}{3a^3\,b}. \tag{45}$$

So, the equilibrium points for the system (40) are $E_1 = (0, 0)$ and $E_2 = (-\frac{\delta_1}{\delta_2}, 0)$. Take the linearized form of the system (40) at equilibrium point (X_0, Y_0) and let

$$M(X_0, Y_0) = \begin{pmatrix} 0 & 1 \\ -\delta_1 & 0 \end{pmatrix}. \tag{46}$$

So, we can analyze the stability of these points by computing the Jacobian matrix $J(X, Y) = \frac{\partial(f,g)}{\partial(X,Y)} = \begin{pmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{pmatrix}_{X=X_0, Y=Y_0}$ of the system (40) as:

$$J(X_0, Y_0) = \begin{vmatrix} 0 & 1 \\ -\delta_1 - 2\delta_2 X & 0 \end{vmatrix} = \delta_1 + 2\delta_2 X. \tag{47}$$

The eigenvalues of J are given by $|J - \lambda I| = 0$ which implies,

$$\lambda^2 - tr(J)\,\lambda + |J| = 0. \tag{48}$$

where $tr(J) = 0$ and $|J| = \delta_1 + 2\delta_2 X$. And, the following deductions are utilized:

- If the jacobian is positive, then the equilibrium point represents a center point.
- If the jacobian is negative, then the equilibrium point represents a saddle point.
- If the jacobian is zero, then the equilibrium point represents a cuspidal point. The parameter ranges can be changed to achieve the anticipated outcomes.

(a): $\delta_1 > 0, \ \delta_2 > 0$. For the parametric values $a = 1, b = 1, c = 2, \alpha = 0.4$ and $\beta = 03$, the equilibrium points are obtained as $E_1 = (0, 0)$ and $E_2 = \left(-\frac{8}{30}, 0\right)$, where E_1 is a center point, since $J(0, 0) = \delta_1 = 0.8 > 0$, and E_2 is saddle point, since $J(-1, 0) = \delta_1 - -2\delta_2 = -5.2 < 0$, as shown in Fig. (11).

(b): $\delta_1 > 0, \ \delta_2 < 0$. For the parametric values $a = -1, b = 1, c = 2, \alpha = 0.4$ and $\beta = 03$, the equilibrium points are obtained as $E_1 = (0, 0)$ and $E_2 = \left(\frac{8}{30}, 0\right)$, where E_1 is a center point, since $J(0, 0) > 0$, and E_2 is saddle point, since $J(-1/2, 0) < 0$, as shown in Fig. (12).

(c): $\delta_1 < 0, \ \delta_2 < 0$. For the parametric values $a = -1, b = 2, c = 1, \alpha = 0.4$ and $\beta = 03$, the equilibrium points are obtained as $E_1 = (0, 0)$ and $E_2 = \left(-\frac{13}{30}, 0\right)$, where E_1 is a saddle point, since $J(0, 0) < 0$, and E_2 is saddle point, since $J(1, 0) < 0$, as shown in Fig. (13).

(d): $\delta_1 < 0, \ \delta_2 > 0$. For the parametric values $a = 1, b = 2, c = 1, \alpha = 0.4$ and $\beta = 03$, the equilibrium points are obtained as $E_1 = (0, 0)$ and $E_2 = \left(\frac{13}{30}, 0\right)$, where E_1 is a saddle point, since $J(0, 0) < 0$, and E_2 is center point, since $J(1, 0) > 0$, as shown in Fig. (14).

7 Numerical simulation of results

The dynamical behavior of achieved results of Date-Jimbo-Kashiwara-Miwa model is designed in Figs. 1, 2, 3, 4, 5, 67. The traveling wave solution of $g_3^1(x,y,t)$, $g_4^1(x,y,t)$, $g_6^1(x,y,t)$, $g_7^1(x,y,t)$, $g_{11}^1(x,y,t)$, $g_{14}^1(x,y,t)$ and $g_1^2(x,y,t)$ are depicted in part (a) whereas part (b) shows the corresponding density plots. Part(c) illustrates line graph for different values of t. Figures 1 and 7 shows periodic traveling waves. Figures 2-6 depict dark, bright and kink solitons. The appropriate values of parameters on every figures are mentioned in caption. Figure 1 depicts the periodic type solution of $g_3^1(x,y,t)$ for the rates of $a = 1,\ a_0 = 0,\ \alpha = 1,\ A = 1,\ b = 1.5,\ \beta = 1,\ B = 0,\ \xi_0 = 1,\ \lambda = 0.125$. Figure 2 depicts the soliton solution

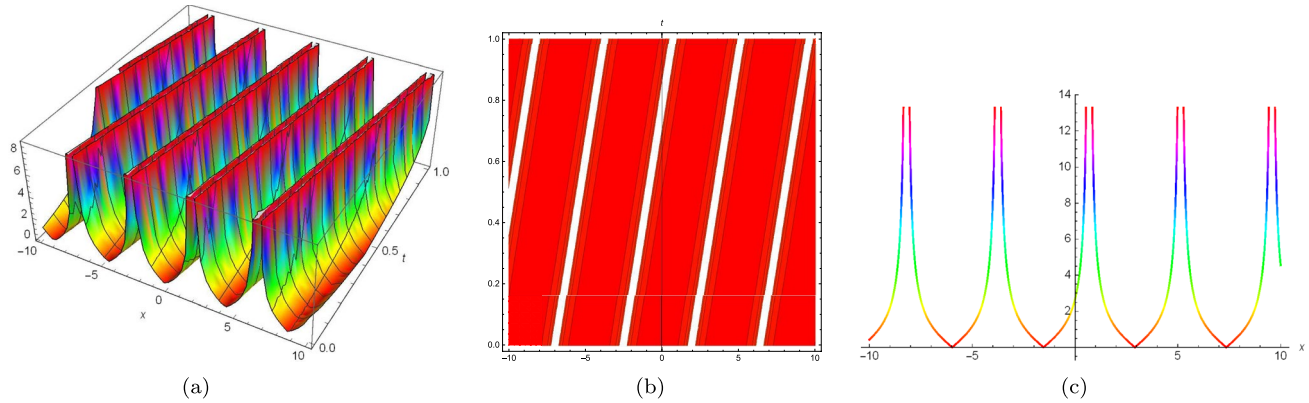

Fig. 1 Plots of the finding $g_3^1(x,y,t)$ of (1) using $a = 1,\ a_0 = 0,\ \alpha = 1,\ A = 1,\ b = 1.5,\ \beta = 1,\ B = 0,\ \xi_0 = 1,\ \lambda = 0.125$

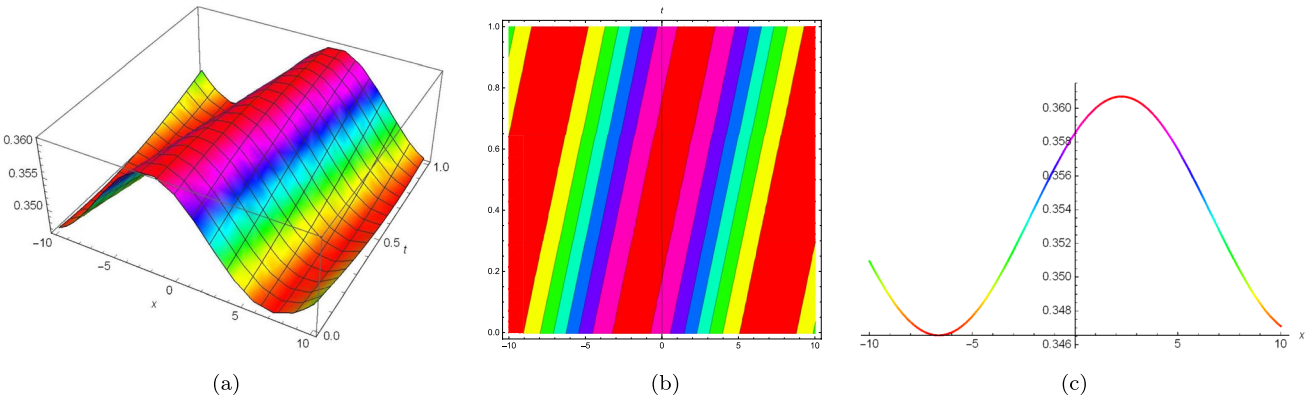

Fig. 2 Plots of the finding $g_4^1(x,y,t)$ of (1) using a=0.25, $a_0 = 0,\ \alpha = 0.5,\ A = 0.1,\ b = 0.5,\ \beta = 0.5,\ \xi_0 = 0,\ \lambda = 0.125$

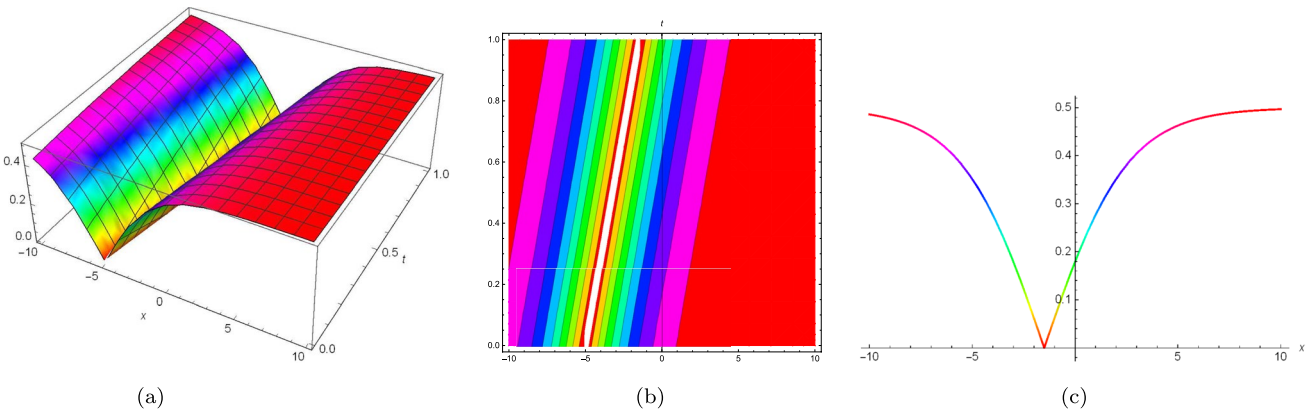

Fig. 3 Plots of the finding $g_6^1(x,y,t)$ of (1) using a=0.5, $a_0 = 0,\ \alpha = 0.5,\ A = 1,$ B=0, $b = 1,\ \beta = 0.5,\ \xi_0 = 1.5,\ \lambda = -0.25$

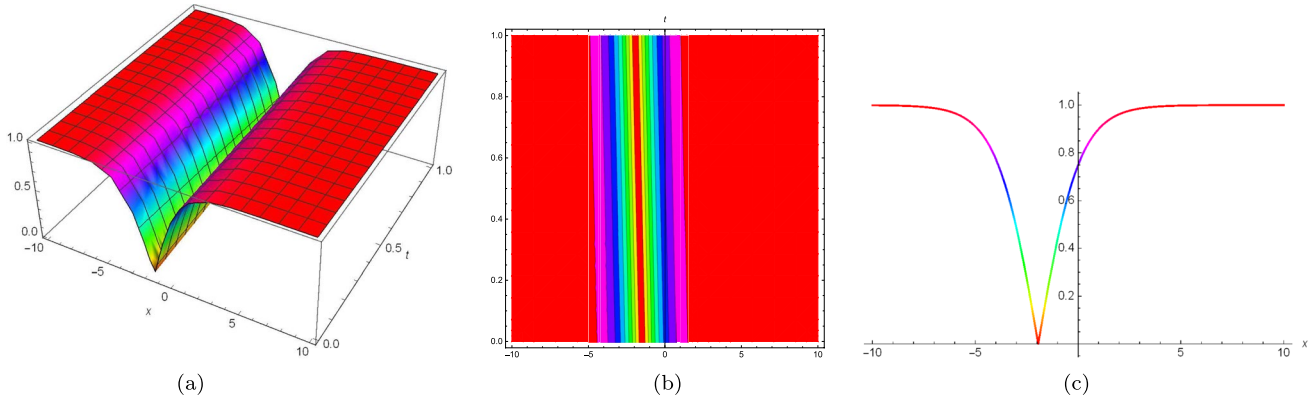

Fig. 4 Plots of the finding $g_7^1(x,y,t)$ of (1) using a=1, $a_0 = 0$, $\alpha = 0.5$, $A = 1$, $b = 0.5$, $\beta = 0.5$, $\xi_0 = 1$, $\lambda = -0.25$

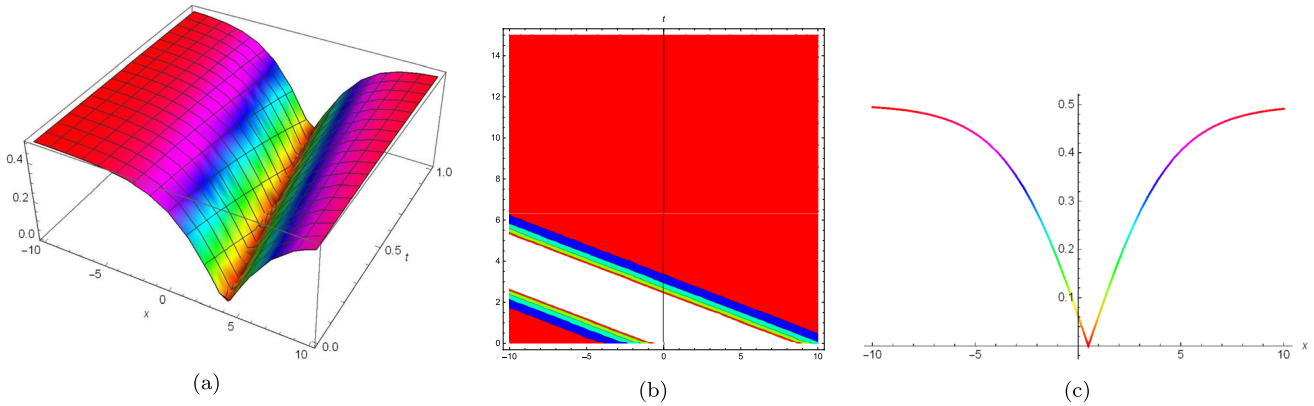

Fig. 5 Plots of the finding $g_{11}^1(x,y,t)$ of (1) using a=−0.5, $a_0 = 0$, $\alpha = 0.5$, $A = 1$, $b = 1$, $\beta = 0.5$, $B = 0$, $\xi_0 = 1$, $\lambda = -0.25$

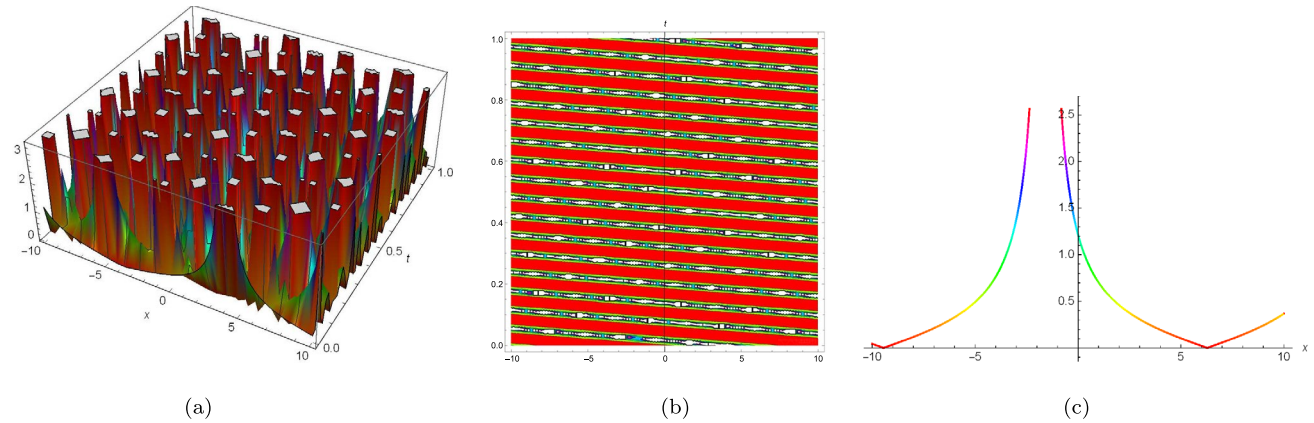

Fig. 6 Plots of the finding $g_{14}^1(x,y,t)$ of (1) using $a = 0.125$, $a_0 = 0$, $\alpha = 0.55$, $A = 1$, $b = -1$, $\beta = 0.55$, $\xi_0 = 0.5$, $\lambda = 2.555$

of $g_4^1(x,y,t)$ for the rates of $a = 0.25$, $a_0 = 0$, $\alpha = 0.5$, $A = 0.1$, $b = 0.5$, $\beta = 0.5$, $\xi_0 = 0$, $\lambda = 0.125$. Figure 3 shows the soliton solution of $g_6^1(x,y,t)$ for the rates of $a = 0.5$, $a_0 = 0$, $\alpha = 0.5$, $A = 1$, B=0, $b = 1$, $\beta = 0.5$, $\xi_0 = 1.5$, $\lambda = -0.25$. Figure 4 shows the kink soliton solution of $g_7^1(x,y,t)$ for the rates of $a = -0.5$, $a_0 = 0$, $\alpha = 0.5$, $A = 1$, $b = 1$, $\beta = 0.5$, $B = 0$, $\xi_0 = 1$, $\lambda = -0.25$. Also, the figure 5 shows the kink soliton solution of $g_{11}^1(x,y,t)$ for the rates of $a = 1$, $a_0 = 0$, $\alpha = 0.5$, $A = 1$, $b = 0.5$, $\beta = 0.5$, $\xi_0 = 1$, $\lambda = -0.25$. Moreover, the figure 6 presents the kink soliton solution of $g_{14}^1(x,y,t)$ for the rates of $a_0 = 0$, $\alpha = 0.55$, $A = 1$, $b = -1$, $\beta = 0.55$, $\xi_0 = 0.5$, $\lambda = 2.555$. In addition, the figure 7 presents the periodic

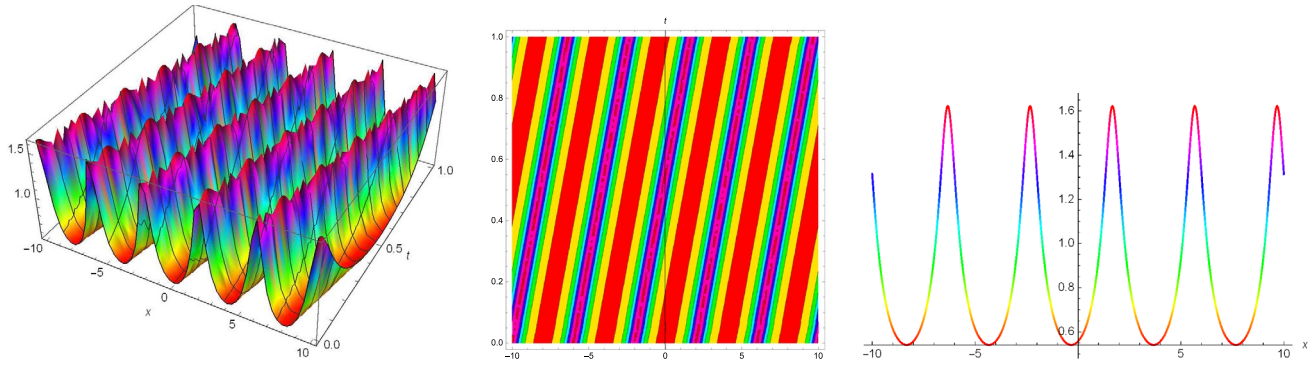

Fig. 7 Plots of the finding $g_1^2(x, y, t)$ of (1) using $k = 2.25,\ k_0 = 0,\ \alpha = 0.5,\ a = -0.5,\ b = -1,\ \beta = 0.5,\ d = 0.5$

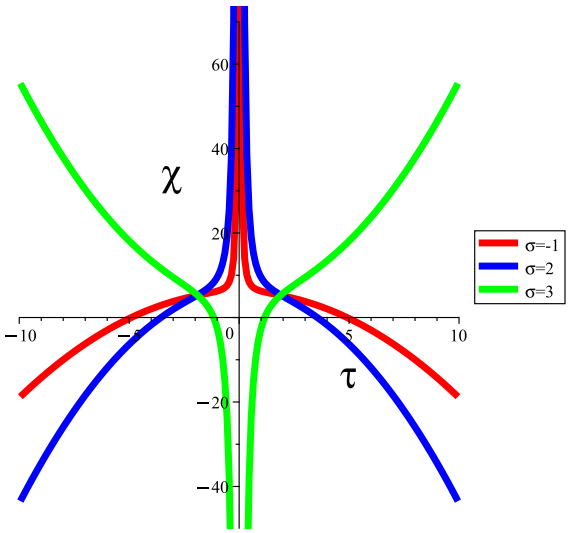

Fig. 8 The dispersion between $\chi(\tau)$ and wave number σ with values $\alpha = 2, \beta = 2, A_0 = -2, B_0 = 3$.

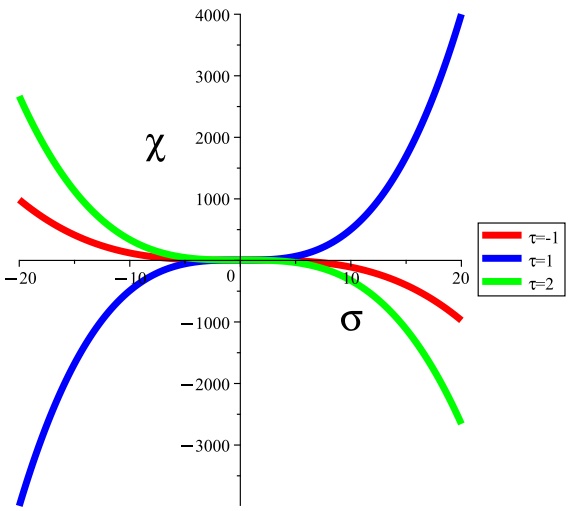

Fig. 9 The dispersion between $\chi(\sigma)$ and wave number τ with values $\alpha = 2, \beta = 2, A_0 = -2, B_0 = 3$.

wave solution of $g_1^2(x, y, t)$ for the rates of $k = 2.25,\ k_0 = 0,\ \alpha = 0.5,\ a = -0.5,\ b = -1,\ \beta = 0.5,\ d = 0.5$. Its solutions have physical interpretations in different contexts. For instance, bright solitons represent localized energy 13 packets in

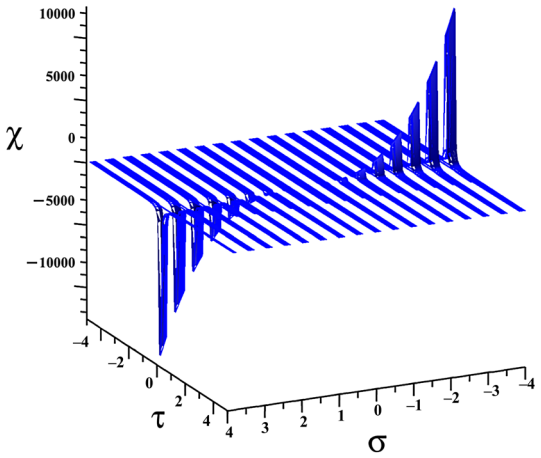

Fig. 10 The dispersion between $\chi(\sigma)$ and wave number τ with values $\alpha = 2, \beta = 2, A_0 = -2, B_0 = 3$.

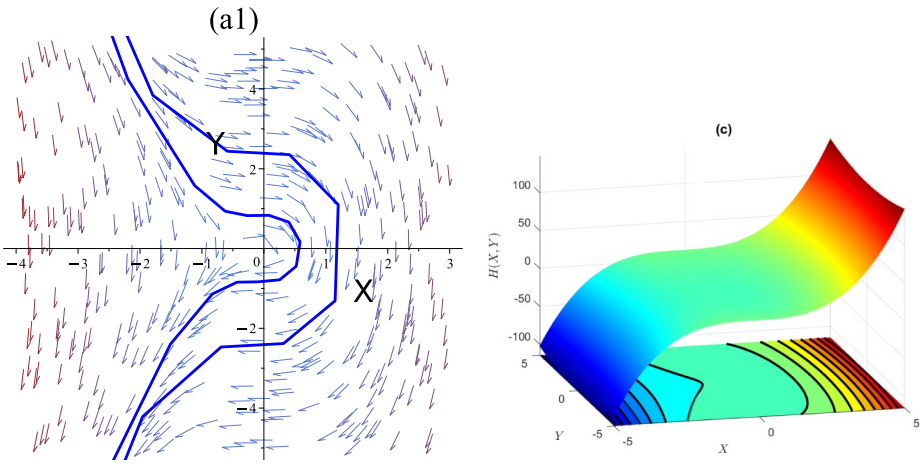

Fig. 11 Bifurcation analysis of planar dynamical system (40) when $\delta_1 > 0,\ \delta_2 > 0$.

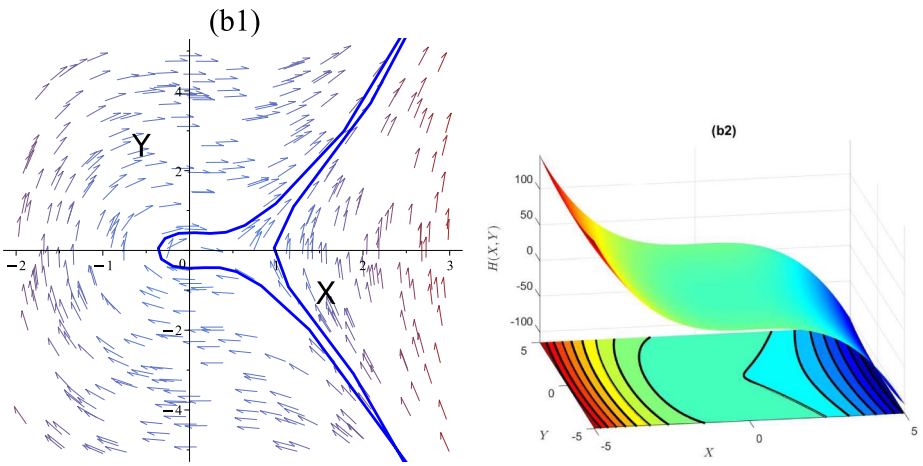

Fig. 12 Bifurcation analysis of planar dynamical system (40) when $\delta_1 > 0,\ \delta_2 < 0$.

nonlinear optics and Bose-Einstein condensates. Dark solitons correspond to regions of lower density in Bose-Einstein condensates and nonlinear 140 optics. Periodic solitons appear in periodic structures such as crystal lattices and periodic optical waveguides. Anti-kink solitons represent a type of topological defect in the field theory. Optical solitons can travel long distances without spreading due to their self-focusing and self-phase modulation properties. The results reveal the existence of different soliton solutions when modeling the weakly nonlinear restoring forces and long water waves with frequency dispersion for studying dynamics of a broad range of physical systems. This equation has been investigated

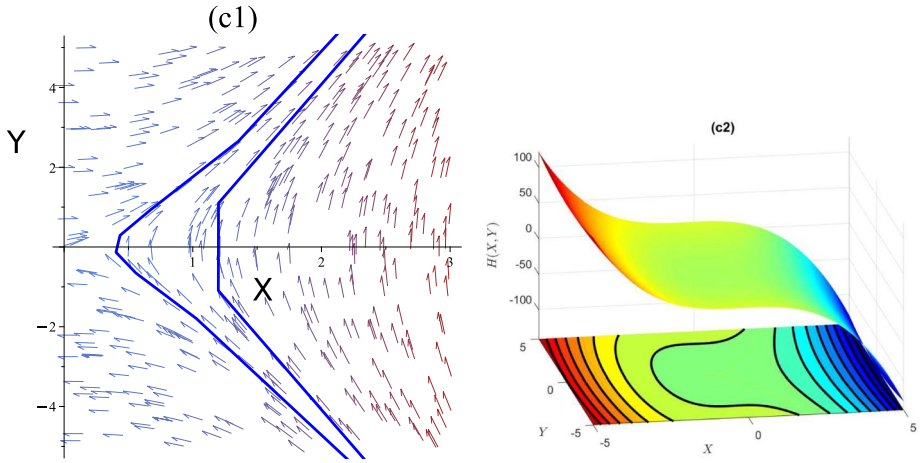

Fig. 13 Bifurcation analysis of planar dynamical system (40) when $\delta_1 < 0,\ \delta_2 < 0$.

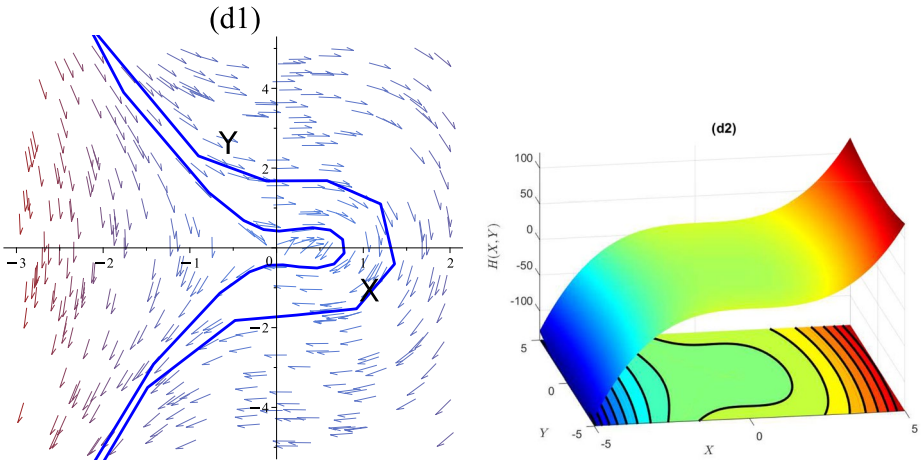

Fig. 14 Bifurcation analysis of planar dynamical system (40) when $\delta_1 < 0,\ \delta_2 > 0$.

in the literature employing a variety of capable techniques within the writing, counting, the improved Bernoulli sub-equation method [46], the two-variable $(\frac{G'}{G}, \frac{1}{G})$-expansion technique [47], the extended direct algebraic method [48], the Lie symmetry method [49], the modified *F*-expansion technique [50] and the finite difference method [51].

8 Conclusion

In our work, the unified method, the modified Kudryashov technique and the extended modified AEM technique were implemented to attain some new traveling wave solutions to the $(2+1)$ dimensional DJKM model. Some solutions were presented graphically to understand their behavior with the help of Mathematica 18, for different sets of parameter. As a result, we acquired the soliton solution, periodic wave solution, kink soliton solution and the singular solution respectively. The stability analysis was investigated. Furthermore, the qualitative behavior of the dynamical system at the equilibrium points has been observed through bifurcation analysis. Moreover, using the help of Maple computer program, we put out the direction of the obtained solutions for visualizing the accomplished dynamical features. The strength of the described equation is that it was predictable, trustworthy and computationally attractive. We are confident that the described methods will play a major role in future research. Despite some limitations, these methods represent the significant advances in the study of nonlinear wave equations, offering a promising approach for future research on higher-order nonlinear systems.

Acknowledgements The authors extend their appreciation to Taif University, Saudi Arabia, for supporting this work through project number (TU-DSPP-2024–51).

Author contributions **Kalim U. Tariq**: Writing, Validation, Formal analysis, Software; **Jalil Manafian**: Supervision, Mathematical analysis, Methodology, Writing-review & editing; **Somaye Malmir**: Investigation, Writing-review & editing; **R. Nadir Tufail**: Formal analysis, Writing-review & editing; **Onur Alp Ilhan**: Writing-review, Software; **Khaled H. Mahmoud**: Resources, Writing-review & editing. All authors have read and agreed to the published version of the manuscript.

Funding This research was funded by Taif University, Saudi Arabia, Project No. (TU-DSPP-2024–51).

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics and Consent to Participate declarations not applicable.

Consent to Publish declaration not applicable.

Conflict of interest The authors declare they have no Conflict of interest regarding the publication of the article.

Open Access This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License, which permits any non-commercial use, sharing, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if you modified the licensed material. You do not have permission under this licence to share adapted material derived from this article or parts of it. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by-nc-nd/4.0/.

References

1. Durand M, Langevin D. Physicochemical approach to the theory of foam drainage. Euro Phys J E. 2002;7(1):35–44.
2. Kudryashov N. On types of nonlinear nonintegrable equations with exact solutions. Phys Lett A. 1991;155(4–5):269–75.
3. Li L-X, Li E-Q, Wang M-L. The (g′/g, 1/g)-expansion method and its application to travelling wave solutions of the zakharov equations. Appl Math-A J Chinese Univ. 2010;25(4):454–62.
4. Hosseini K, Mayeli P, Kumar D. New exact solutions of the coupled sine-gordon equations in nonlinear optics using the modified kudryashov method. J Mod Opt. 2018;65(3):361–4.
5. Wazwaz A-M. Gaussian solitary waves for the logarithmic-kdv and the logarithmic-kp equations. Phys Scr. 2014;89(9): 095206.
6. Ivezić Ž, Kahn SM, Tyson JA, Abel B, Acosta E, Allsman R, Alonso D, AlSayyad Y, Anderson SF, Andrew J, et al. Lsst: from science drivers to reference design and anticipated data products. Astrophys J. 2019;873(2):111.
7. Bekir A, Boz A. Exact solutions for nonlinear evolution equations using exp-function method. Phys Lett A. 2008;372(10):1619–25.
8. Bekir A. Application of the (g/ g)-expansion method for nonlinear evolution equations. Phys Lett A. 2008;372(19):3400–6.
9. Osman M. Multi-soliton rational solutions for quantum zakharov-kuznetsov equation in quantum magnetoplasmas. Waves Randomand Complex Media. 2016;26(4):434–43.
10. Seadawy AR, Cheemaa N. Applications of extended modified auxiliary equation mapping method for high-order dispersive extended nonlinear schrödinger equation in nonlinear optics. Mod Phys Lett B. 2019;33(18):1950203.
11. Naher H, Abdullah FA. The modified benjamin-bona-mahony equation via the extended generalized riccati equation mapping method. Appl Math Sci. 2012;6(111):5495–512.
12. Wazwaz A-M. A sine-cosine method for handlingnonlinear wave equations. Math Comput Model. 2004;40(5–6):499–508.
13. Ahmed I, Seadawy AR, Lu D. M-shaped rational solitons and their interaction with kink waves in the fokas-lenells equation. Phys Scr. 2019;94(5): 055205.
14. Kudryashov NA. The painlevé approach for finding solitary wave solutions of nonlinear nonintegrable differential equations. Optik. 2019;183:642–9.
15. Wazwaz A-M. The extended tanh method for abundant solitary wave solutions of nonlinear wave equations. Appl Math Comput. 2007;187(2):1131–42.
16. Sun Y-L, Ma W-X, Yu J-P. N-soliton solutions and dynamic property analysis of a generalized three-component hirota-satsuma coupled kdv equation. Appl Math Lett. 2021;120: 107224.
17. Takougang E, Ali M, Bouzidi Y, Bouchaala F, Sultan A, Mohamed A. Extraction and characterization of faults and fractures from 3d vsp data in a carbonate reservoir: A workflow. J Petrol Sci Eng. 2019;182: 106328.
18. Bouchaala F, Ali M, Farid A. Estimation of compressional seismic wave attenuation of carbonate rocks in abu dhabi, united arab emirates. comptes rendus, Géoscience 346(7-8) (2014) 169–178.
19. Ugli TBB, Zaurbekov N. Modeling unmanned submarine by an adaptive control to follow an ellipse path by considering disturbances on the system. J Res Sci, Eng Tech. 2022;10(4):173–87.
20. Alkhazaleh HA, Pallathadka H. Optimizing the maintenance and repair program based on condition monitoring in combined cycle power plants. J Res Sci, Eng Tech. 2022;10(3):157–72.

21. Younis M, Ali S, Rizvi STR, Tantawy M, Tariq KU, Bekir A. Investigation of solitons and mixed lump wave solutions with (3+ 1)-dimensional potential-ytsf equation. Commun Nonlinear Sci Numer Simul. 2021;94: 105544.
22. Ullah MS, Ali MZ, Roshid H-O, Seadawy A, Baleanu D. Collision phenomena among lump, periodic and soliton solutions to a (2+ 1)-dimensional bogoyavlenskii's breaking soliton model. Phys Lett A. 2021;397: 127263.
23. El-Tantawy S, Salas AH, Alharthi M. Novel analytical cnoidal and solitary wave solutions of the extended kawahara equation. Chaos, Solitons & Fractals. 2021;147: 110965.
24. Park C, Nuruddeen R, Ali KK, Muhammad L, Osman M, Baleanu D. Novel hyperbolic and exponential ansatz methods to the fractional fifth-order korteweg-de vries equations. Adv Differ Equ. 2020;2020(1):1–12.
25. Sarwar S, Furati KM, Arshad M. Abundant wave solutions of conformable space-time fractional order fokas wave model arising in physical sciences. Alex Eng J. 2021;60(2):2687–96.
26. Zhang J-L, Wang M-L, Wang Y-M, Fang Z-D. The improved f-expansion method and its applications. Phys Lett A. 2006;350(1–2):103–9.
27. Mahak N, Akram G. The modified auxiliary equation method to investigate solutions of the perturbed nonlinear schrödinger equation with kerr law nonlinearity. Optik. 2020;207: 164467.
28. Khater MM, Lu D, Attia RA. Dispersive long wave of nonlinear fractional wu-zhang system via a modified auxiliary equation method. AIP Adv. 2019;9(2): 025003.
29. Khater M, Attia RA, Lu D. Modified auxiliary equation method versus three nonlinear fractional biological models in present explicit wave solutions. Math Comput Appl. 2019;24(1):1.
30. Ahmed ZJ, Mustfai MA. Exploration of vehicle target detection and classification method based on sea lion optimization with deep convolutional neural network. J Smart Internet Things. 2023;2022(1):65–80.
31. Alsalami Z. Modeling of optimal fully connected deep neural network based sentiment analysis on social networking data. J Smart Internet Things. 2023;2022(1):114–32.
32. Mohammed GG, Zaheer Z. Neurocyberguard: Developing a robust cybersecurity defense system through deep neural learning-based mathematical modeling. J Smart Internet Things. 2023;2022(1):133–45.
33. Ali KK, Mehanna MS, Wazwaz A-M. Analytical and numerical treatment to the (2+ 1)-dimensional date-jimbo-kashiwara-miwa equation. Nonlinear Eng. 2021;10(1):187–200.
34. Arnous AH, Biswas A, Ekici M, Alzahrani AK, Belic MR. Optical solitons and conservation laws of kudryashov's equation with improved modified extended tanh-function. Optik. 2021;225: 165406.
35. Gasimov YS, Nachaoui A, Niftiyev AA. On the regularization cauchy problem for matrix factorizations of the helmholtz equation in a multidimensional bounded domain. Azerbaijan J Math. 2022;12(1):142–61.
36. Ekici M, Sonmezoglu A. Optical solitons with biswas-arshed equation by extended trial function method. Optik. 2019;177:13–20.
37. Biswas A, Ekici M, Sonmezoglu A, Belic MR. Solitons in optical fiber bragg gratings with dispersive reflectivity by extended trial function method. Optik. 2019;182:88–94.
38. Nawaz B, Rizvi STR, Ali K, Younis M. Optical soliton for perturbed nonlinear fractional schrödinger equation by extended trial function method. Opt Quant Electron. 2018;50(5):1–14.
39. Ugli TBB, Zaurbekov N, Nurdaulet D, Anzum R. Design and control of a rehabilitation arm robot to regulate the patient's activity using the integrated force control and fuzzy logic. J Res Sci, Eng Tech. 2022;10(3):134–42.
40. Lu D, Seadawy A, Arshad M. Applications of extended simple equation method on unstable nonlinear schrödinger equations. Optik. 2017;140:136–44.
41. Zayed EM, Shohib RM. Optical solitons and other solutions to biswas-arshed equation using the extended simplest equation method. Optik. 2019;185:626–35.
42. Almusawa H, Ali K. K, Wazwaz A.-M, Mehanna M, Baleanu D, Osman M. Protracted study on a real physical phenomenon generated by media inhomogeneities, Results in Physics (2021) 104933.
43. Khan K, Akbar MA. The exp (- ϕ (ξ))-expansion method for finding travelling wave solutions of vakhnenko-parkes equation. Int J Dyn Syst Differ Equ. 2014;5(1):72–83.
44. Khater MM, Mousa A, El-Shorbagy M, Attia RA. Analytical and semi-analytical solutions for phi-four equation through three recent schemes. Res Phys. 2021;22: 103954.
45. Alderremy AA, Attia RA, Alzaidi JF, Lu D, Khater M. Analytical and semi-analytical wave solutions for longitudinal wave equation via modified auxiliary equation method and adomian decomposition method. Therm Sci. 2019;23(Suppl. 6):1943–57.
46. Kumar A, Ilhan E, Ciancio A, Yel G, Baskonus HM. Extractions of some new travelling wave solutions to the conformable date-jimbo-kashiwara-miwa equation. AIMS Math. 2021;6(5):4238–64.
47. Iqbal MA, Wang Y, Miah MM, Osman MS. Study on date-jimbo-kashiwara-miwa equation with conformable derivative dependent on time parameter to find the exact dynamic wave solutions. Fractal Fr. 2022;6(1):4.
48. Akram G, Sajid N, Abbas M, Hamed Y, Abualnaja K. M. Optical solutions of the date–jimbo–kashiwara–miwa equation via the extended direct algebraic method, Journal of Mathematics 2021.
49. Bibi K, Ahmad K. New exact solutions of date jimbo kashiwara miwa equation using lie symmetry groups. Math Probl Eng. 2021;2021:1–8.
50. Seadawy A. R, Ali A. Stability analysis and abundant closed-form wave solutions of the date–jimbo–kashiwara–miwa and combined sinh–cosh-gordon equations arising in fluid mechanics, International Journal of Nonlinear Sciences and Numerical Simulation.
51. ŞENER S. Ş. New wave behaviors of the (3+ 1)-dimensional date-jimbo-kashiwara-miwa equation, Journal of Mathematical Sciences and Modelling 4 (3) 126–132.
52. Alquran M. Variation of the influence of atangana-conformable time-derivative on various physical structures in the fractional kp-bbm model. Int J Theor Phys. 2024;63:225.
53. Al-Shara M, Alquran H, Jaradat I. Jaradat, Analysis of optical bi-wave solutions in a two-mode model arising from the unstable schrödinger equation. Int J Theor Phys. 2024;63:88.
54. Alquran M. New interesting optical solutions to the quadratic-cubic schrodinger equation by using the kudryashov-expansion method and the updated rational sine-cosine functions. Opt Quant Electron. 2022;54:666.

55. Alquran M. Physical properties for bidirectional wave solutions to a generalized fifth-order equation with third-order time-dispersion term. Res Phys. 2021;28: 104577.
56. Liu Y, Manafian J, Singh G, Alkader NA, Nisar KS. Analytical investigations of propagation of ultra-broad nonparaxial pulses in a birefringent optical waveguide by three omputational ideas. Sci Rep 2024;14:6317
57. Wen S, Manafian J, Sedighi S, Atmaca SP, Gallegos C, Mahmoud KH, Alsubaie AS. Interactions among lump optical solitons for coupled nonlinear schrödinger equation with variable coefcient via bilinear method. Sci Rep 2024;14:19568
58. Li W, Kuang Y, Manafian J, Malmir S, Eslami B, Mahmoud KH, Alsubaie AS. Multiple rogue wave, double-periodic soliton and breather wave solutions for a generalized breaking soliton system in (3+1)-dimensions. Sci Rep 2024;14:19723.
59. Luo J, Manafian J, Eslami B, Mahmoud KH, Sharma R, Kumari N, Alsubaie AS. Assorted optical solitons of the (1+1)- and (2+1)-dimensional chiral nonlinear schrödinger equations using modified extended tanh-function technique. Sci Rep. 2024;14:25530.
60. Guo Y, Chen Y, Manafian J, Malmir S, Mahmoud KH, Alsubaie AS. Exploring n-soliton solutions, multiple rogue wave and the linear superposition principle to the generalized hirota satsuma-ito equation. Sci Rep. 2024;14:26171.
61. Chen H, Manafian J, Eslami B, Salazar M, Kumari N, Sharma R, Joshi SK, Mahmoud KH, Alsubaie AS. Lump, lump-periodic, lump-soliton and multi soliton solutions for the potential kadomtsev-petviashvili type coupled system with variable coefficients. Sci Rep. 2024;14:27682.
62. Shao R, Manafian J, Ilhan OA, Mahmoud KH, AbdAlreda B, Alsubaie AS. Computational study of thin films made from the ferroelectric materials with paul painlevé approach and expansion and variational methods. Sci Rep. 2024;14:29039.
63. Zou Q, Manafian J, Malmir S, Mahmoud KH, Alsubaie AS, Ewadh NA, Alrekabi I. Exact breather waves solutions in a spatial symmetric nonlinear dispersive wave model in (2+1)-dimensions. Sci Rep. 2024;14:31718.
64. Rao X, Manafian J, Gavahi M, Eslami B, Abdulameer M. K, Alwail Q. A, Enas R, Qahraman Qizi, Paul-painlevé analysis, soliton and periodic wave in the fractional thermophoretic motion equation via graphene sheets, Qualitative Theory of Dynamical Systems 23 (2024) 136.
65. Qawaqneh H, Manafian J, Alsubaie AS, Ahmad H. Investigation of exact solitons to the quartic rosenau-kawahara-regularized-long-wave fluid model with fractional derivative and qualitative analysis. Phys Scr. 2025;100: 015270.
66. Yang Q, Manafian J, Mahmoud KH, Aldurayhim A. Abundant exact traveling wave solutions and modulation instability analysis to the generalized hirota-satsuma-ito equation. Open Phys. 2024;22:20240027.
67. Pradhan B, Boro A. N, Dev Birbaishri, Manafian J, Alkader N. A. Effect of ion anisotropy pressure in viscous plasmas: evolution of shock wave, Nonlinear Dynamics 122 (2024) 17403–17416.
68. Zhang J, Manafian J, Raut S, Roy S, Mahmoud KH, Alsubaie ASA. Study of two soliton and shock wave structures by weighted residual method and hirota bilinear approach. Nonlinear Dyn. 2024;112:12375–91.
69. Wang K-J, Shi F, Li S, Xu P. Dynamics of resonant soliton, novel hybrid interaction, complex n-soliton and the abundant wave solutions to the (2+1)-dimensional boussinesq equation. Alex Eng J. 2024;105:485–95.
70. Wang K-J. The generalized (3+1)-dimensional b-type kadomtsev-petviashvili equation: resonant multiple soliton, n-soliton, soliton molecules and the interaction solutions. Nonlinear Dyn. 2024;112:7309–24.
71. Wang K.-J, Wang G.-D, Shi F, Liu X.-L, Zhu H.-W. Variational principle, hamiltonian, bifurcation analysis, chaotic behaviors and the diverse solitary wave solutions of the simplified modified camassa-holm equation, International Journal of Geometric Methods in Modern Physics.
72. Wang K-J, Li S. Novel complexiton solutions to the new extended (3+1)-dimensional boiti-leon-manna-pempinelli equation for incompressible fluid. Europhys Lett. 2024;146(6):62003.
73. Wang K.-J, Liu J.-H. Mathematical model and the solution of the capillary vibration in a nanoscale deformable, Mathematical Methods in the Applied Sciences.
74. Fokas A, Lenells J. The unified method: I. nonlinearizable problems on the half-line, Journal of Physics A: Mathematical and Theoretical 45 (19) (2012) 195201.
75. Ege SM, Misirli E. The modified kudryashov method for solving some fractional-order nonlinear equations. Adv Difference Equ. 2014;2014(1):1–13.
76. Al-Munawarah A-M, Arabia S. Optical soliton perturbation with fractional temporal evolution by extended modified auxiliary equation mapping. Rev Mex Fis. 2021;67(3):403–14.
77. Guo F, Lin J. Interaction solutions between lump and stripe soliton to the (2+ 1)-dimensional date-jimbo-kashiwara-miwa equation. Nonlinear Dyn. 2019;96(2):1233–41.
78. Darweesh A, Alquran M. New algorithm for deriving rational form solutions in the generalized date-jimbo-kashiwara-miwa model. Phys Scr. 2024;99(10): 105271.
79. Wang K, Zhu H, Shi F, Liu X, Wang G, Li G. Lump wave, breather wave and other abundant wave solutions to the (2+1)-dimensional sawada-kotera-kadomtsev petviashvili equation of fluid mechanics. Pramana. 2025;99:40.
80. Liang Y, Wang K. Bifurcation analysis, chaotic phenomena, variational principle, hamiltonian, solitary and periodic wave solutions of the fractional benjamin ono equation. Fractals. 2025;33(01):2550016.
81. Wang K, Li S, Wang G, Xu P, Shi F, Liu X. Localized wave and other special wave solutions to the (3+1)-dimensional kudryashov-sinelshchikov equation, Mathematical Methods in the Applied Sciences.
82. Wang K, Liu X, Shi F, Li G. Bifurcation and sensitivity analysis, chaotic behaviors, variational principle, hamiltonian and diverse wave solutions of the new extended integrable kadomtsev-petviashvili equation. Phys Lett A. 2025;534: 130246.
83. Wang K, Shi F, Li S, Xu P. Dynamics of resonant soliton, novel hybrid interaction, complex n-soliton and the abundant wave solutions to the (2+1)-dimensional boussinesq equation. Alex Eng J. 2024;105:485–95.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.