EK 1 –

Örnek: $\displaystyle\int_0^{\infty} \frac{dx}{(1+x^2)(1+\arctan x)}$ integralinin değerini hesaplayınız. Yakınsak veya ıraksak olup olmadığını belirtiniz.

Çözüm: 1. tip improper integraldir.

$$\int_0^{\infty} \frac{dx}{(1+x^2)(1+\arctan x)} = \lim_{b\to\infty} \int_0^{b} \frac{dx}{(1+x^2)(1+\arctan x)}$$

$1+\arctan x = t$ dersek $\frac{1}{1+x^2}dx = dt$ olur.

$$= \lim_{b\to\infty} \int_1^{1+\arctan b} \frac{dt}{t}$$

$$= \lim_{b\to\infty} \ln t \Big|_1^{1+\arctan b} = \lim_{b\to\infty} \ln|1+\arctan b| - \underbrace{\ln 1}_{0}$$

$= \ln|1+\frac{\pi}{2}|$ bulunur. Yakınsaktır.

Örnek: $\displaystyle\int_{-\infty}^{\infty} e^{-|x|}dx = ?\ \lim_{a\to-\infty}\int_a^0 e^{-|x|}dx + \lim_{b\to\infty}\int_0^b e^{-|x|}dx$

$$= \lim_{a\to-\infty}\int_a^0 e^{x}dx + \lim_{b\to\infty}\int_0^b e^{-x}dx$$

$$= \lim_{a\to-\infty} e^{x}\Big|_a^0 + \lim_{b\to\infty} -e^{-x}\Big|_0^b$$

$$= \lim_{a\to-\infty} (1-\underbrace{e^{a}}_{0}) + \lim_{b\to\infty} (-\underbrace{e^{-b}}_{0}+1) = 1+1 = 2$$ olur. Yakınsaktır.

EK-2

Örnek: $\displaystyle\int_1^{\infty}\frac{1}{x\sqrt{x^2-1}}\,dx$ integralini hesaplayınız.

Çözüm:

$$\int_1^{\infty}\frac{1}{x\sqrt{x^2-1}}\,dx=\underbrace{\int_1^{c}\frac{1}{x\sqrt{x^2-1}}\,dx}_{\text{2. Tip İmproper İntegrali}}+\underbrace{\int_c^{\infty}\frac{1}{x\sqrt{x^2-1}}\,dx}_{\text{1. Tip İmproper İntegrali}},\quad c>1$$

$c=2$ olsun.

$$=\int_1^{2}\frac{1}{x\sqrt{x^2-1}}\,dx+\int_2^{\infty}\frac{1}{x\sqrt{x^2-1}}\,dx$$

$$=\lim_{a\to1^+}\int_a^{2}\frac{1}{x\sqrt{x^2-1}}\,dx+\lim_{b\to\infty}\int_2^{b}\frac{1}{x\sqrt{x^2-1}}\,dx$$

$$\boxed{x=\sec t},\quad dx=\sec t\tan t\,dt$$

$$=\lim_{a\to1^+}\int\frac{\cancel{\sec t}\cdot\tan t\,dt}{\cancel{\sec t}\cdot\underbrace{\sqrt{\sec^2t-1}}_{\tan^2t}}+\lim_{b\to\infty}\int\frac{\sec t\tan t\,dt}{\sec t\cdot\underbrace{\sqrt{\sec^2t-1}}_{\tan^2t}}$$

$$=\lim_{a\to1^+}\int dt+\lim_{b\to\infty}\int dt=\lim_{a\to1^+}t\Big|+\lim_{b\to\infty}t\Big|$$

$$=\lim_{a\to1^+}\operatorname{arcsec}x\Big|_a^2+\lim_{b\to\infty}\operatorname{arcsec}x\Big|_2^b$$

$$=\lim_{a\to1^+}(\operatorname{arcsec}2-\operatorname{arcsec}a)+\lim_{b\to\infty}(\operatorname{arcsec}b-\operatorname{arcsec}2)=$$

$$=\left(\frac{\pi}{3}-0\right)+\left(\frac{\pi}{2}-\frac{\pi}{3}\right)=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}\text{ bulunur.}$$