YILDIZ TEKNİK ÜNİVERSİTESİ

ELEKTRİK MÜHENDİSLİĞİ BÖLÜMÜ

DEVRE TEORİSİ

Ders Notu-4

AC VE DC DEVRELERDE
YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ

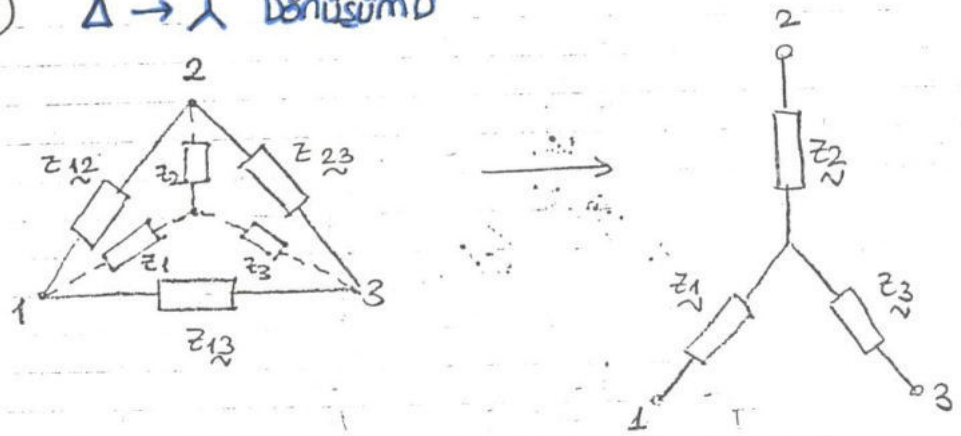
Doç. Dr. Recep YUMURTACI

YILDIZ-ÜGGEN DÖNÜŞÜMLERİ

Δ : Üçgen bağlantı

λ : yıldız //

① Δ → λ Dönüşümü



$\underset{\sim}{z}_{12}, \underset{\sim}{z}_{23}, \underset{\sim}{z}_{13}$ biliniyor.
Bunların λ bağlı eşdeğerleri

$\underset{\sim}{z}_1 = ?$ $\underset{\sim}{z}_2 = ?$ $\underset{\sim}{z}_3 = ?$

$$\underset{\sim}{z}_1 = \frac{\underset{\sim}{z}_{12} \cdot \underset{\sim}{z}_{13}}{\underset{\sim}{z}_{12} + \underset{\sim}{z}_{13} + \underset{\sim}{z}_{23}}$$

$$\underset{\sim}{z}_2 = \frac{\underset{\sim}{z}_{12} \cdot \underset{\sim}{z}_{23}}{\underset{\sim}{z}_{12} + \underset{\sim}{z}_{13} + \underset{\sim}{z}_{23}}$$

$$\underset{\sim}{z}_3 = \frac{\underset{\sim}{z}_{13} \cdot \underset{\sim}{z}_{23}}{\underset{\sim}{z}_{12} + \underset{\sim}{z}_{13} + \underset{\sim}{z}_{23}}$$

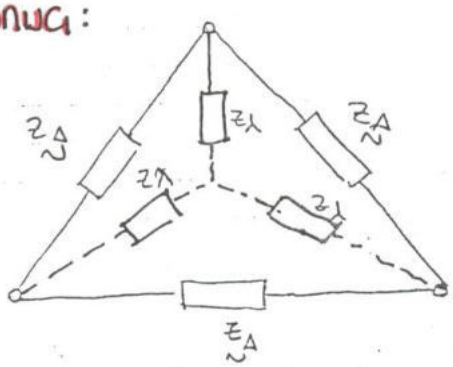
Özel durum

$\underset{\sim}{z}_{12} = \underset{\sim}{z}_{13} = \underset{\sim}{z}_{23} = \underset{\sim}{z}$ olsun.

$$\underset{\sim}{z}_1 = \frac{\underset{\sim}{z} \cdot \underset{\sim}{z}}{3\underset{\sim}{z}} = \frac{\underset{\sim}{z}}{3} \qquad \boxed{\underset{\sim}{z}_1 = \frac{\underset{\sim}{z}}{3}}$$

$$\boxed{\underset{\sim}{z}_2 = \frac{\underset{\sim}{z}}{3}} \qquad \boxed{\underset{\sim}{z}_3 = \frac{\underset{\sim}{z}}{3}}$$

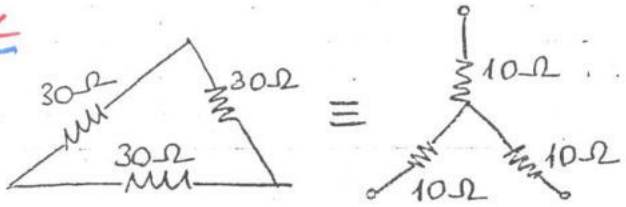
Sonuç:



$$\boxed{\underset{\sim}{z}_\lambda = \frac{\underset{\sim}{z}_\Delta}{3}}$$ #

Not: Bu formüller direnç ve reaktanslar için geçerlidir.

Örnek



$\underset{\sim}{Z}_\Delta = 9 \angle 60°$ olsun.

$\underset{\sim}{Z}_\lambda = \frac{\underset{\sim}{Z}_\Delta}{3} = \frac{9}{3} \angle 60° = 3 \angle 60° \ \Omega$

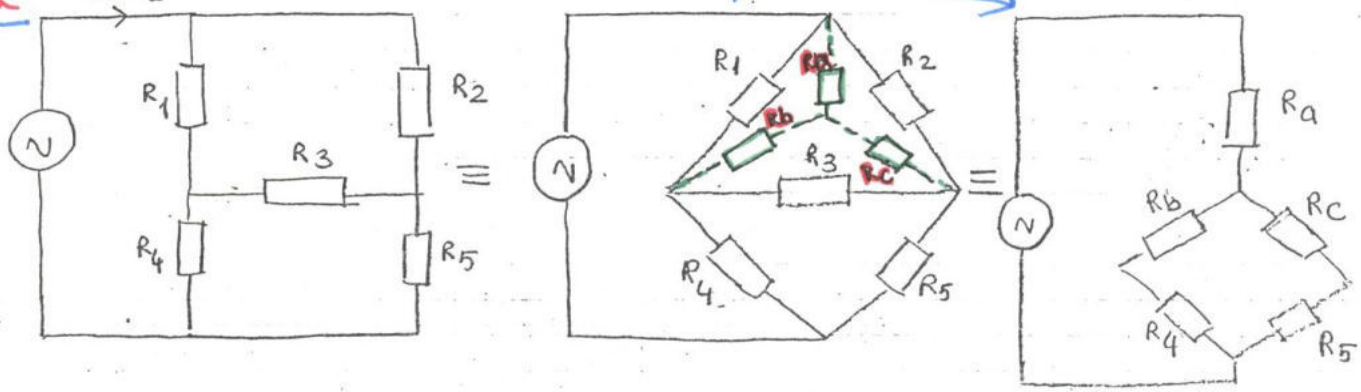
$R_\Delta = 30\ \Omega$

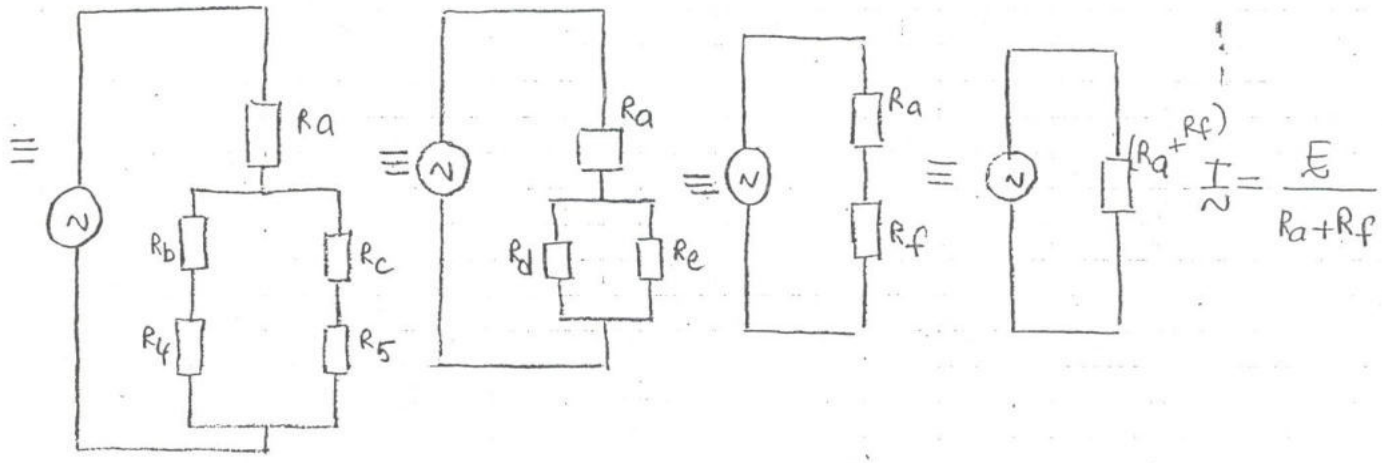
$R_\lambda = \frac{R_\Delta}{3} = 10\ \Omega$

Δ bağlı R_1, R_2, R_3'ü λ'a dönüştürelim

Örnek

$Z = ?$

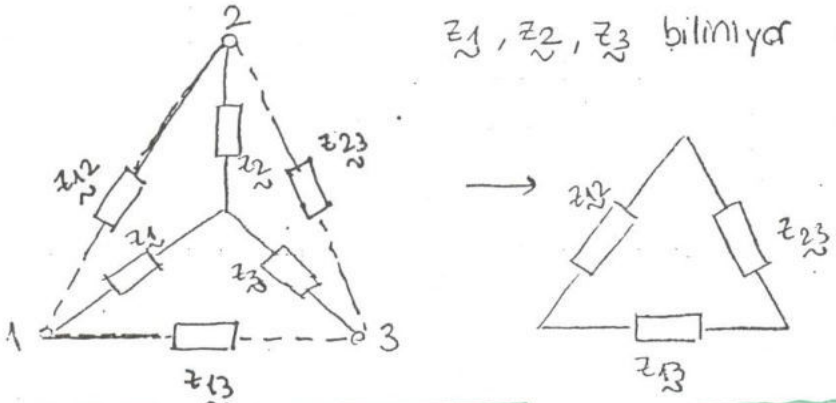




$\underset{\sim}{I} = \frac{E}{R_a + R_f}$

② $\lambda \rightarrow \Delta$ Dönüşümü

$\underset{\sim}{Z}_1, \underset{\sim}{Z}_2, \underset{\sim}{Z}_3$ biliniyor $\underset{\sim}{Z}_{12} = ?$, $\underset{\sim}{Z}_{23} = ?$, $\underset{\sim}{Z}_{13} = ?$

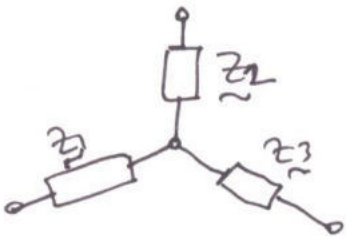


$$\underset{\sim}{Z}_{12} = \frac{\underset{\sim}{Z}_1 \cdot \underset{\sim}{Z}_2 + \underset{\sim}{Z}_1 \cdot \underset{\sim}{Z}_3 + \underset{\sim}{Z}_2 \cdot \underset{\sim}{Z}_3}{\underset{\sim}{Z}_3}$$

$$\underset{\sim}{Z}_{13} = \frac{\underset{\sim}{Z}_1 \underset{\sim}{Z}_3 + \underset{\sim}{Z}_1 \underset{\sim}{Z}_2 + \underset{\sim}{Z}_2 \cdot \underset{\sim}{Z}_3}{\underset{\sim}{Z}_2}$$

$$Z_{23} = \frac{\underset{\sim}{Z}_1 \cdot \underset{\sim}{Z}_2 + \underset{\sim}{Z}_1 \cdot \underset{\sim}{Z}_3 + \underset{\sim}{Z}_2 \cdot \underset{\sim}{Z}_3}{Z_1}$$

Y → Δ Dönüşümünde Özel Durum :



$\underset{\sim}{Z_1} = \underset{\sim}{Z_2} = \underset{\sim}{Z_3} = \underset{\sim}{Z_Y}$ olsun (Empedanslar dengeli)

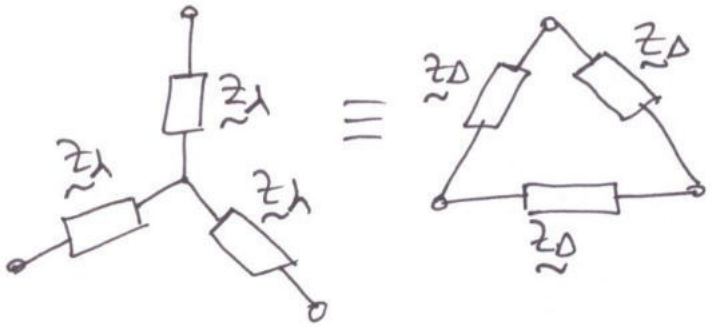
$$\underset{\sim}{Z_{12}} = \frac{3.\underset{\sim}{Z_Y}.\underset{\sim}{Z_Y}}{\underset{\sim}{Z_Y}} = 3.\underset{\sim}{Z_Y}$$

$$\underset{\sim}{Z_{23}} = \frac{3.\underset{\sim}{Z_Y}.\underset{\sim}{Z_Y}}{\underset{\sim}{Z_Y}} = 3.\underset{\sim}{Z_Y}$$

$$\underset{\sim}{Z_{13}} = \frac{3.\underset{\sim}{Z_Y}.\underset{\sim}{Z_Y}}{\underset{\sim}{Z_Y}} = 3.\underset{\sim}{Z_Y}$$

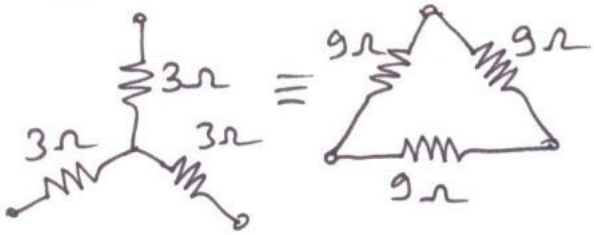
$\Rightarrow \underset{\sim}{Z_{12}} = 3.\underset{\sim}{Z_Y}$, $\underset{\sim}{Z_{23}} = 3.\underset{\sim}{Z_Y}$, $\underset{\sim}{Z_{13}} = 3.\underset{\sim}{Z_Y}$

Sonuç : $\underset{\sim}{Z_1} = \underset{\sim}{Z_2} = \underset{\sim}{Z_3} = \underset{\sim}{Z_Y}$ ise $\underset{\sim}{Z_{12}} = \underset{\sim}{Z_{23}} = \underset{\sim}{Z_{13}} = \underset{\sim}{Z_\Delta} = 3.\underset{\sim}{Z_Y}$ olur.

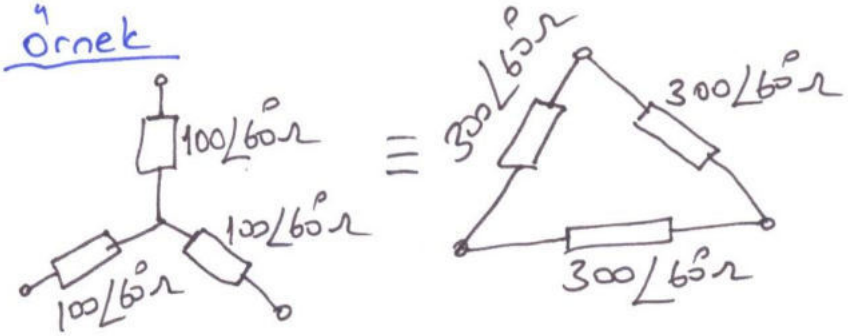


$$\underset{\sim}{Z_\Delta} = 3.\underset{\sim}{Z_Y}$$

1. Örnek

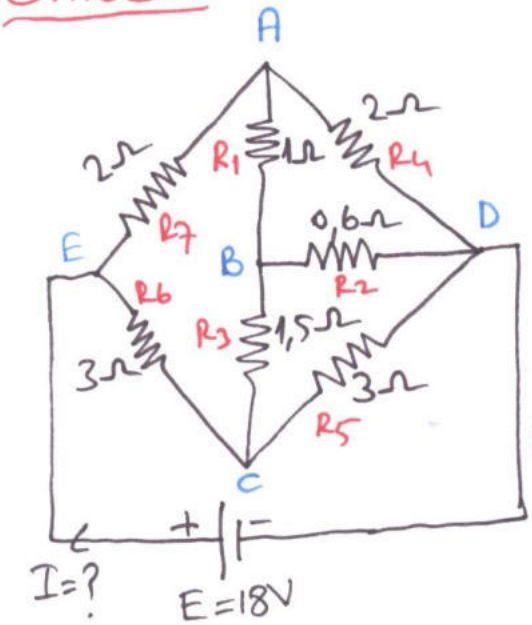


2. Örnek



1. Örnek

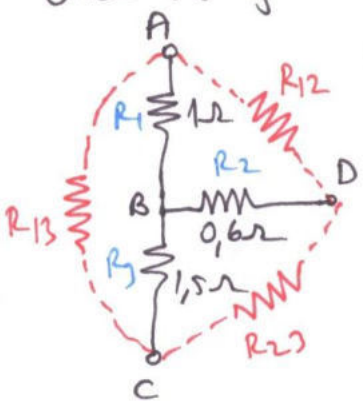
Şekildeki devrede kaynaktan çekilen akımı hesaplayınız



Çözüm

Önce Y bağlı R_1, R_2, R_3'ü Δ'e dönüştürelim

$(R_1.R_2 + R_1.R_3 + R_2.R_3) = 1.0,6 + 1.1,5 + 0,6.1,5 = 3\,\Omega^2$



$$R_{12} = \frac{3}{R_3} = \frac{3}{1,5} = 2\,\Omega$$

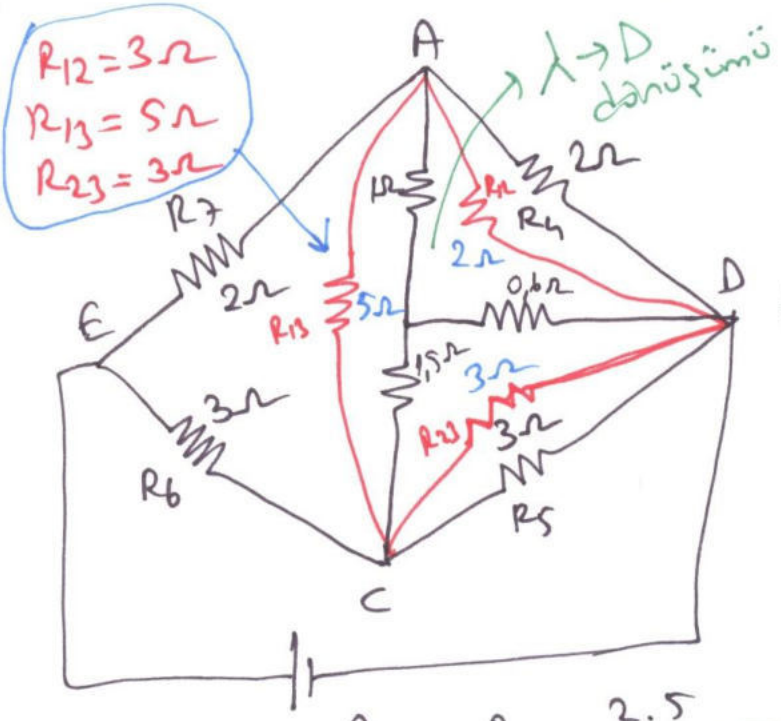
$$R_{13} = \frac{3}{R_2} = \frac{3}{0,6} = 5\,\Omega$$

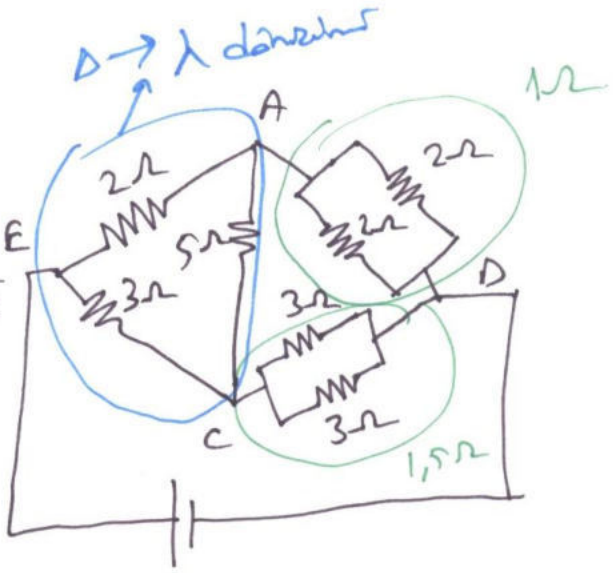
$$R_{23} = \frac{3}{R_1} = \frac{3}{1} = 3\,\Omega$$

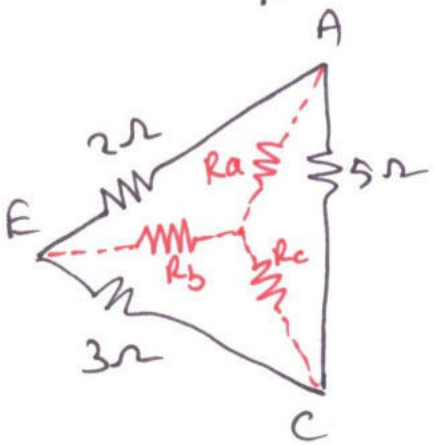
$R_{12} = 3\Omega$

$R_{13} = 5\Omega$

$R_{23} = 3\Omega$



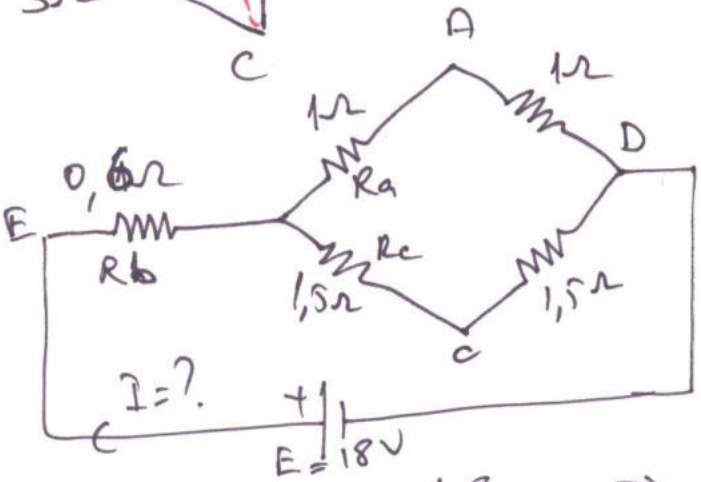




$$R_a = \frac{2.5}{2+3+5} = 1\Omega$$

$$R_b = \frac{2.3}{2+3+5} = 0{,}6\Omega$$

$$R_c = \frac{3.5}{2+3+5} = 1{,}5\Omega$$



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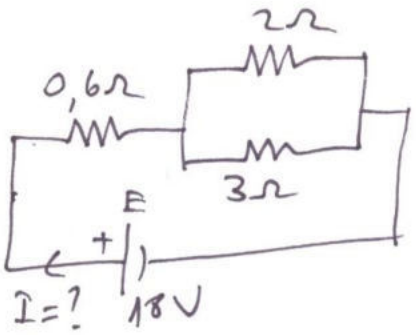
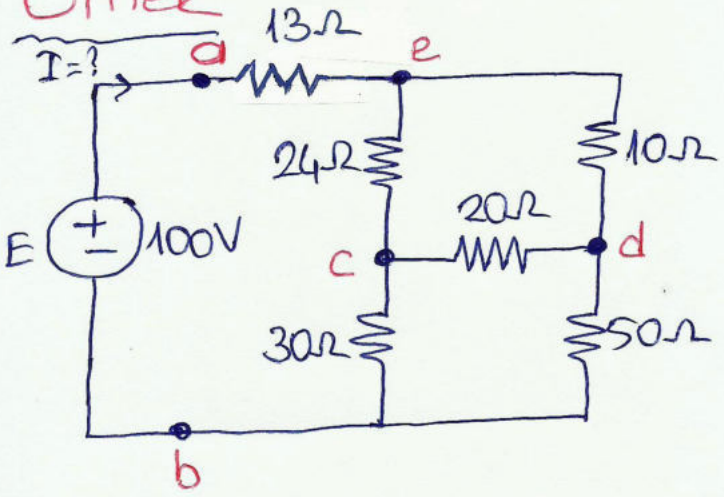


$$R_T = 0{,}6 + \frac{2.3}{2+3} = 1{,}8\Omega \Rightarrow I = \frac{E}{R_T} = \frac{18}{1{,}8} = 10A$$

$\boxed{I = 10A}$

DC DEVRELERDE YILDIZ-ÜÇGEN (λ-Δ) DÖNÜŞÜMLERİ

Örnek



a) R_{ab} = ? ($\Delta \rightarrow \lambda$ dönüşümüyle bulunur)

b) I = ?

c) 13Ω'luk direncin gücünü ve kWh olarak bir ayda harcadığı elektrik enerjisini hesaplayınız.

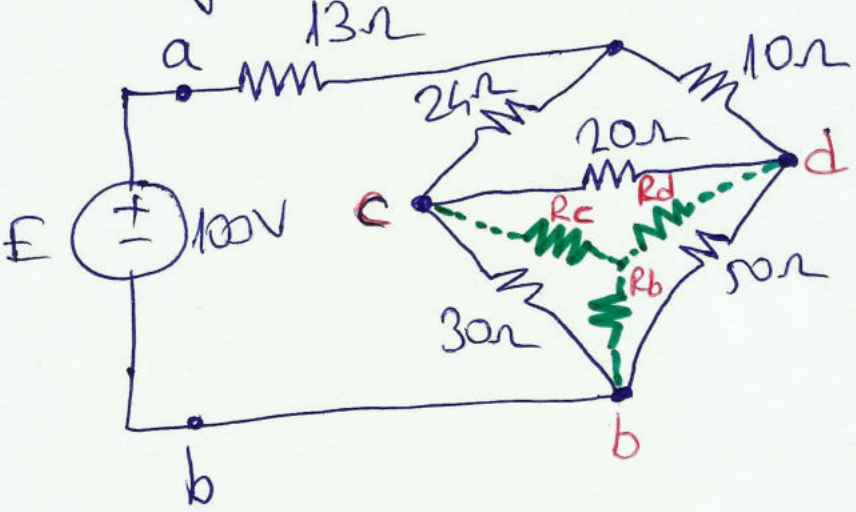
d) Rab direncini 20Ω, 10Ω ve 50Ω ($\lambda_{bağlı}$) dirençlerini Δ'e çevirerek $\Delta \rightarrow \lambda$ dönüşümüyle bulunuz

Çözüm

a) a-b uçları arasındaki eşdeğer direnci bulmak için aşağıdaki λ-Δ dönüşümü seçeneklerinden birini bulabiliriz.

- Δ bağlı olan 20Ω, 30Ω ve 50Ω'u λ'a çevirmek
- " " " 24Ω, 20Ω ve 10Ω'u " "
- λ bağlı olan 20Ω, 10Ω ve 50Ω'u Δ'e çevirmek
- λ " " 24Ω, 20Ω ve 30Ω'u " "

Biz Δ bağlı olan alttaki 20Ω, 30Ω ve 50Ω'luk dirençlere $\Delta \rightarrow \lambda$ dönüşümü uygulayıp bunlara eşdeğerlerini bulalım:

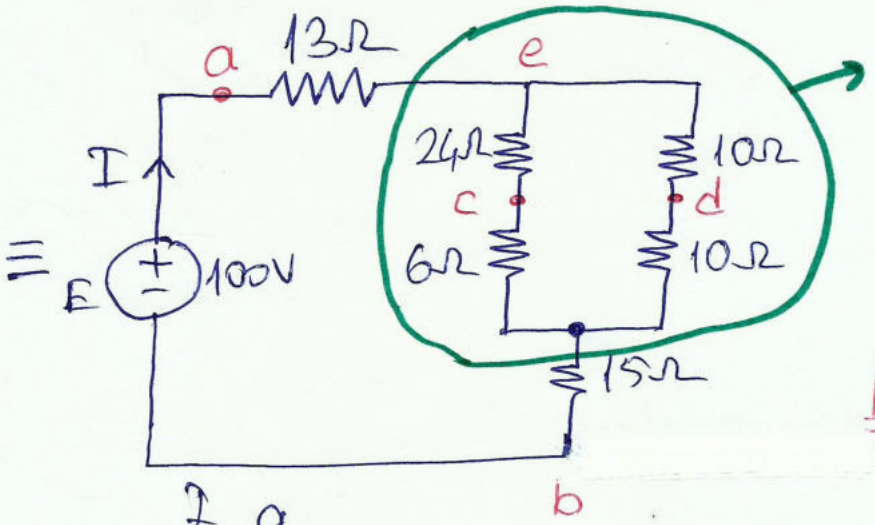


$\sum R_\Delta = 20+30+50 = 100\Omega$

$R_c = \frac{30 \cdot 20}{100} = 6\Omega$

$R_d = \frac{20 \cdot 50}{100} = 10\Omega$

$R_b = \frac{30 \cdot 50}{100} = 15\Omega$

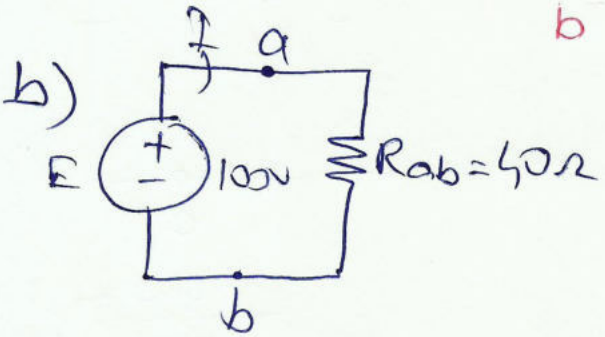


$$Re_1 = \frac{30.20}{30+20} = 12\Omega$$

$$R_{ab} = 13+12+15$$

$$\boxed{R_{ab} = 40\Omega}$$

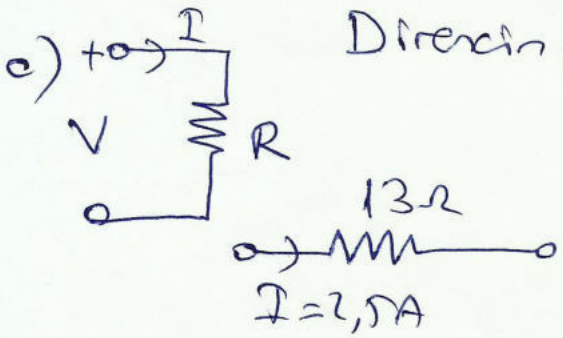
b)



$$I = \frac{E}{R_{ab}} = \frac{100}{40}$$

$$\boxed{I = 2,5A}$$

c)



Direnç için: $\boxed{P = V.I} = (R.I).I \Rightarrow \boxed{P = RI^2}$

$$P = V.I = V.\left(\frac{V}{R}\right) \Rightarrow \boxed{P = \frac{V^2}{R}}$$

$$P = RI^2 = 13.2,5^2$$

$$\boxed{P = 81,25W}$$

Enerji (kWh) olarak

$E = P.t$

↓ kwh ↓ kw ↳ h

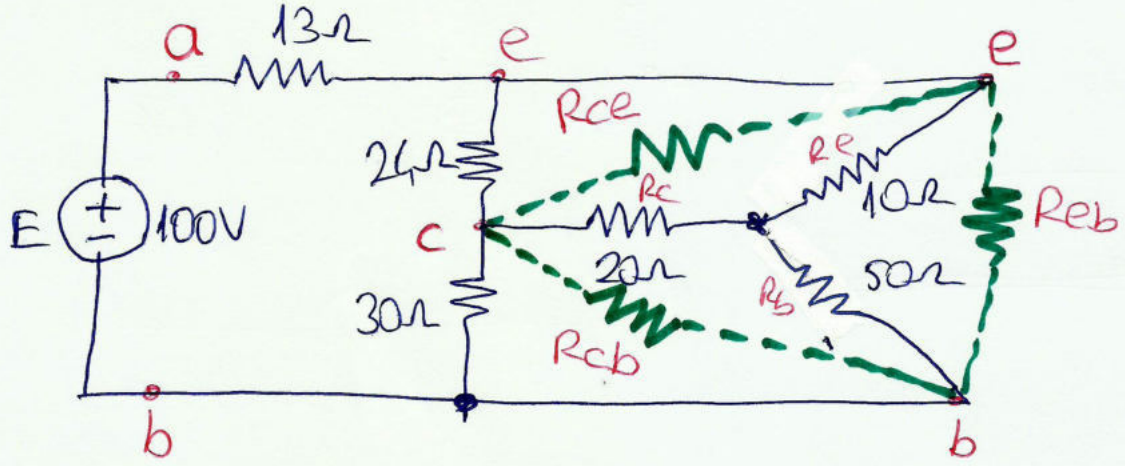
$P = 81,25W = 81,25.10^{-3}kW$

$t = 1ay = 30.24 = 720h$ (saat)

$E = P.t = 81,25.10^{-3}.720$

$$\boxed{E = 58,5\ kWh}$$

d) R_{ab}'nin $\lambda \rightarrow \Delta$ dönüşümüyle bulunması



İkizerli λ bağlı direçlerin çarpımlarının toplamı:

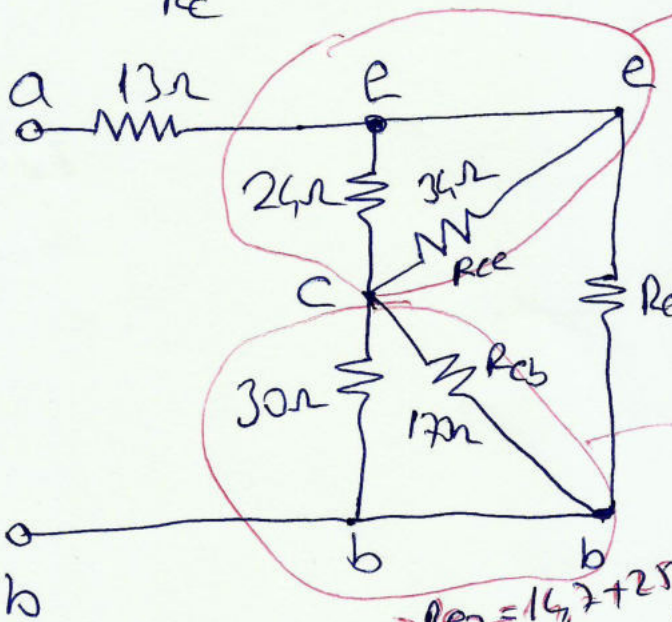
$$R_c.R_b + R_c.R_e + R_b.R_e = 20.50 + 20.10 + 50.10 = 1700$$

$$R_{ce} = \frac{R_c.R_b + R_c.R_e + R_b.R_e}{R_b} = \frac{1700}{50} = 34\Omega$$

$$R_{cb} = \frac{1700}{R_e} = \frac{1700}{10} = 170\Omega$$

$$R_{eb} = \frac{1700}{R_c} = \frac{1700}{20} = 85\Omega$$

$R_{e1} = \frac{24.34}{24+34} = 14{,}07\Omega$

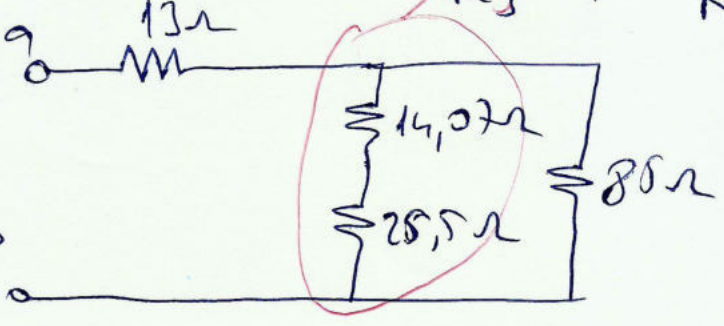


$R_{e2} = \frac{30.170}{30+170} = 25{,}5\Omega$

$R_{eş} = 14{,}7 + 25{,}5 =$

$R_{ab} = 13 + \left(\frac{39{,}57 \cdot 85}{39{,}57 + 85}\right)$

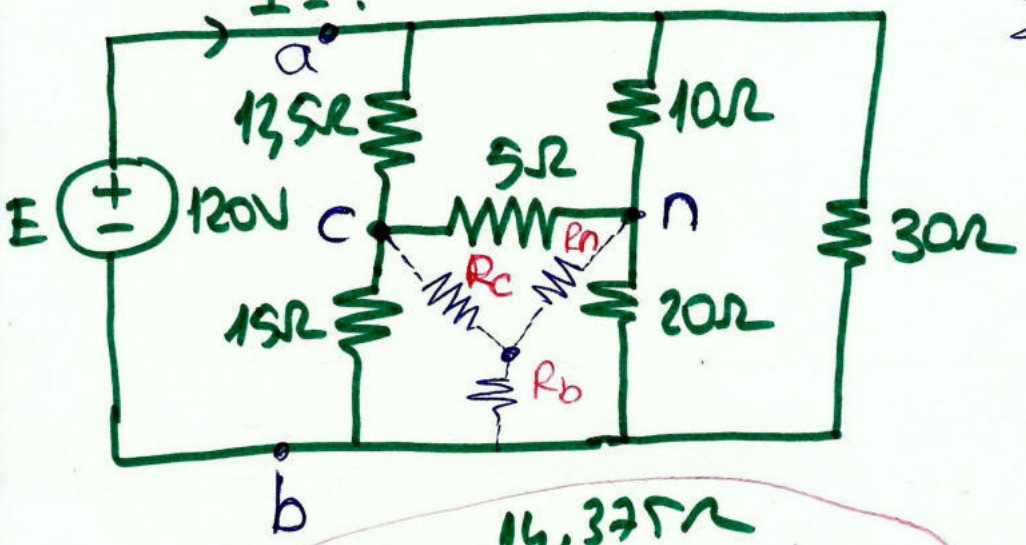
27



$\boxed{R_{ab} = 40\Omega}$

Örnek

Devrede $R_{ab}=?$ $I=?$

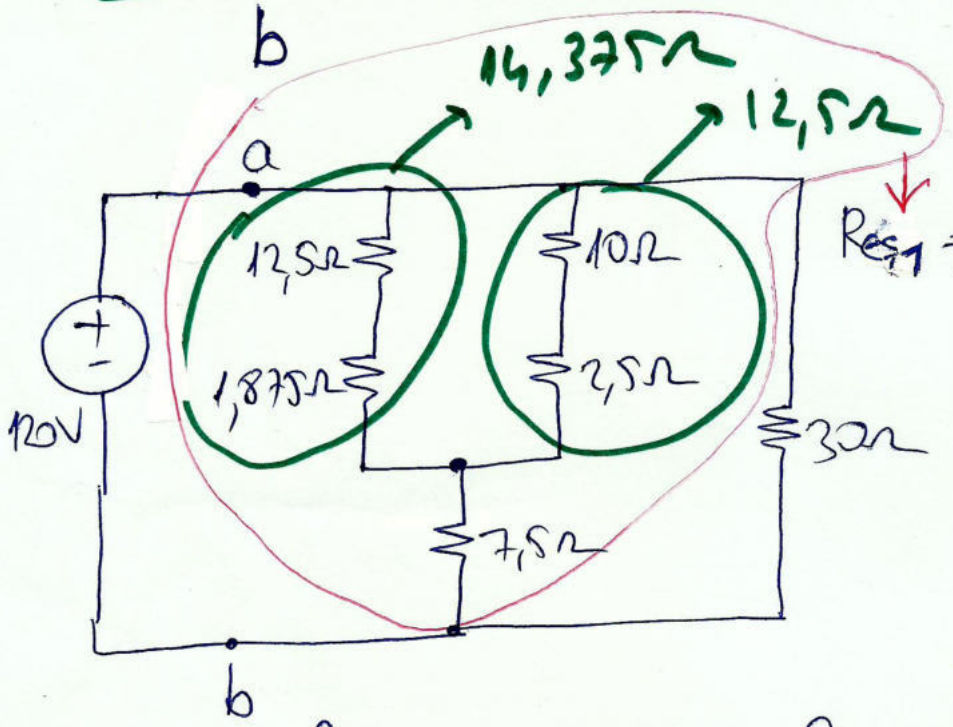


$$\Sigma R_D = 5+15+20 = 40\Omega$$

$$R_c = \frac{15.5}{\Sigma R_D} = \frac{15.5}{40} = 1,375\Omega$$

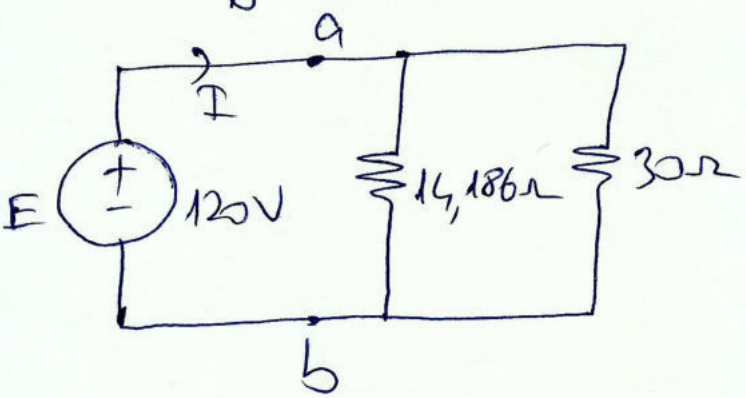
$$R_n = \frac{5.20}{40} = 2,5\Omega$$

$$R_b = \frac{15.20}{40} = 7,5\Omega$$



$$Res_1 = \frac{14,375.12,5}{(14,375+12,5)} + 7,5$$

$$Res_1 = 14,186\Omega$$

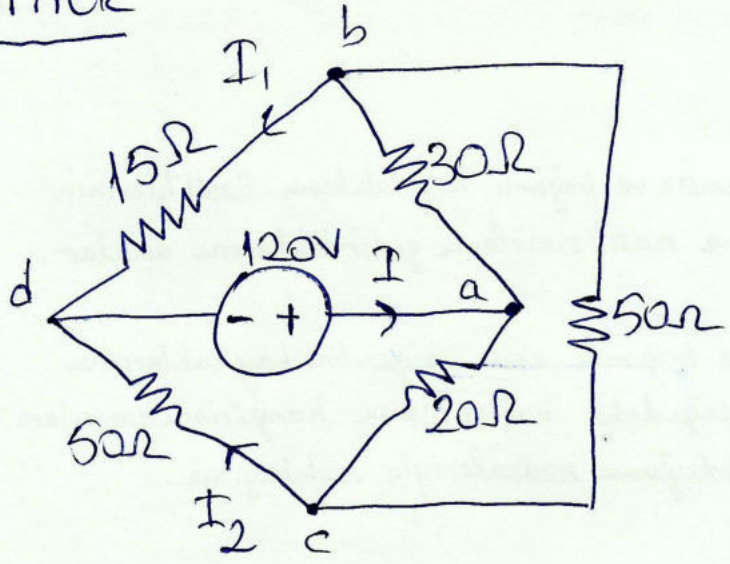


$$R_{ab} = \frac{14,186.30}{(14,186+30)}$$

$$\boxed{R_{ab} = 9,632\Omega}$$

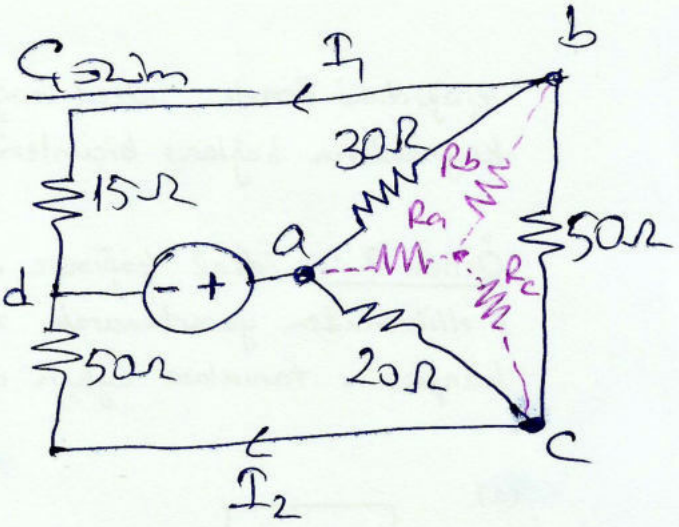
$$I = \frac{E}{R_{ab}} = \frac{120}{9,632} \Rightarrow \boxed{I = 12,458A}$$

Örnek



$I_t = ?,\ I_1 = ?,\ I_2 = ?$

Çözüm



Δ–Y Dönüşümü

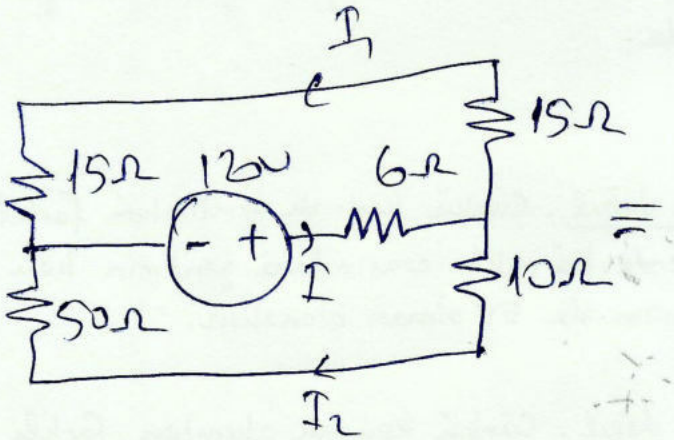
$$\sum R_\Delta = 30+20+50 = 100\Omega$$

$$R_a = \frac{30\cdot 20}{100} = 6\Omega$$

$$R_b = \frac{30\cdot 50}{100} = 15\Omega$$

$$R_c = \frac{20\cdot 50}{100} = 10\Omega$$

≡



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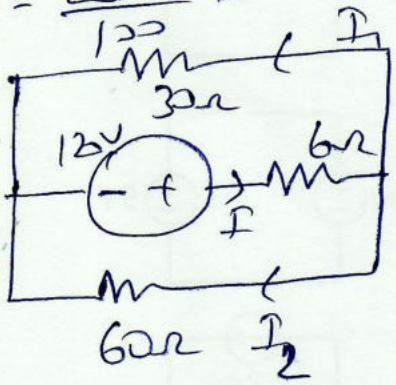

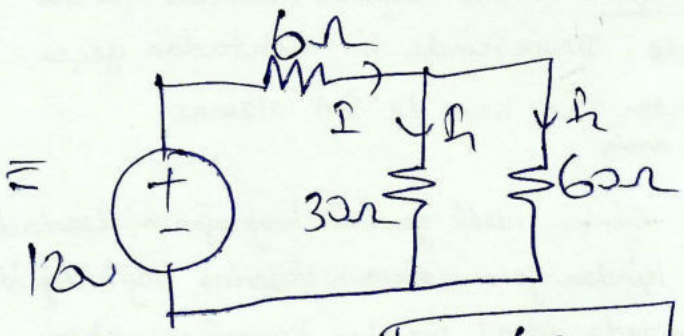

≡

$$R_T = 6 + \frac{30\cdot 60}{30+60} = 26\Omega$$

$$\boxed{R_T = 26\Omega}$$

$$I = \frac{120}{26} \Rightarrow \boxed{I = \frac{60}{13}\,A}$$

Akım bölücüden: $I_1 = \frac{60}{30+60}\cdot I = \frac{60}{90}\cdot\frac{60}{13} \Rightarrow \boxed{I_1 = \frac{40}{13}\,A}$

$$I_2 = \frac{30}{90}\cdot\frac{60}{13} \Rightarrow \boxed{I_2 = \frac{20}{13}\,A}$$