BEARING CAPACITY OF SHALLOW FOUNDATIONS

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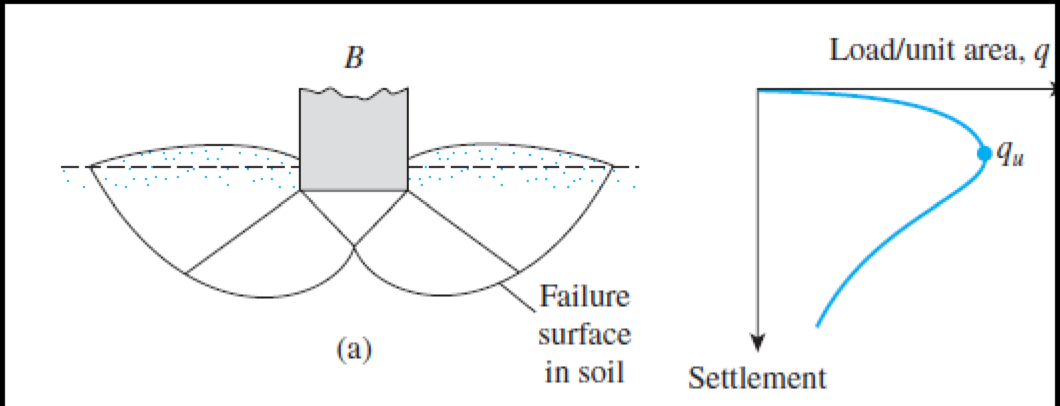
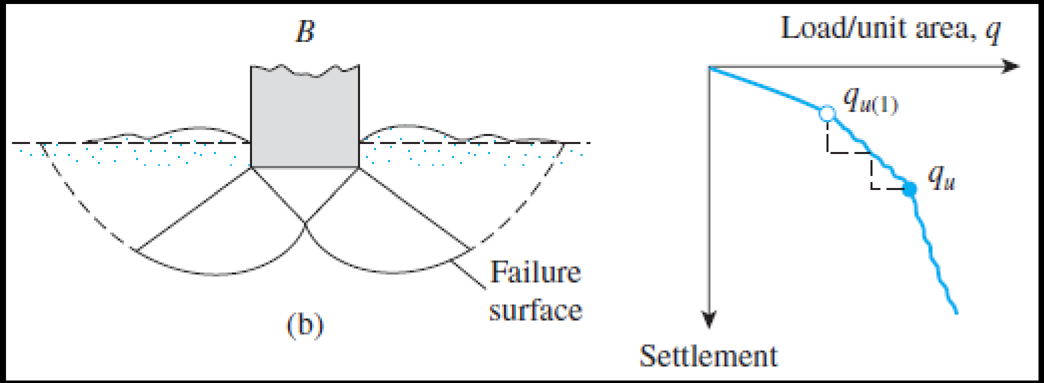
- The lowest part of a structure is generally referred to as the *foundation.*
- *The load per unit area of the foundation at which*
- Its function is to transfer the load of the structure to the soil on which it is resting.
- To perform satisfactorily, shallow foundations must have two main charactersitic:

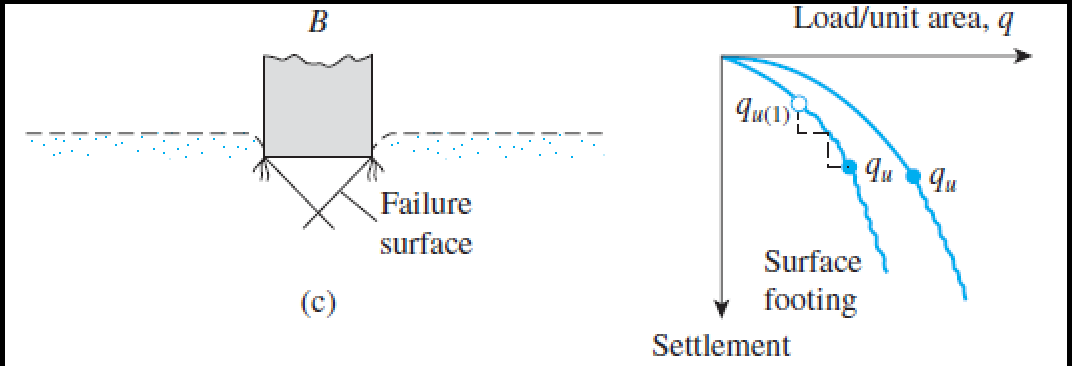
1. They have to be safe against overall shear failure in the soil that supports them.
2. They cannot undergo excessive displacement or settlement.



general shear failure

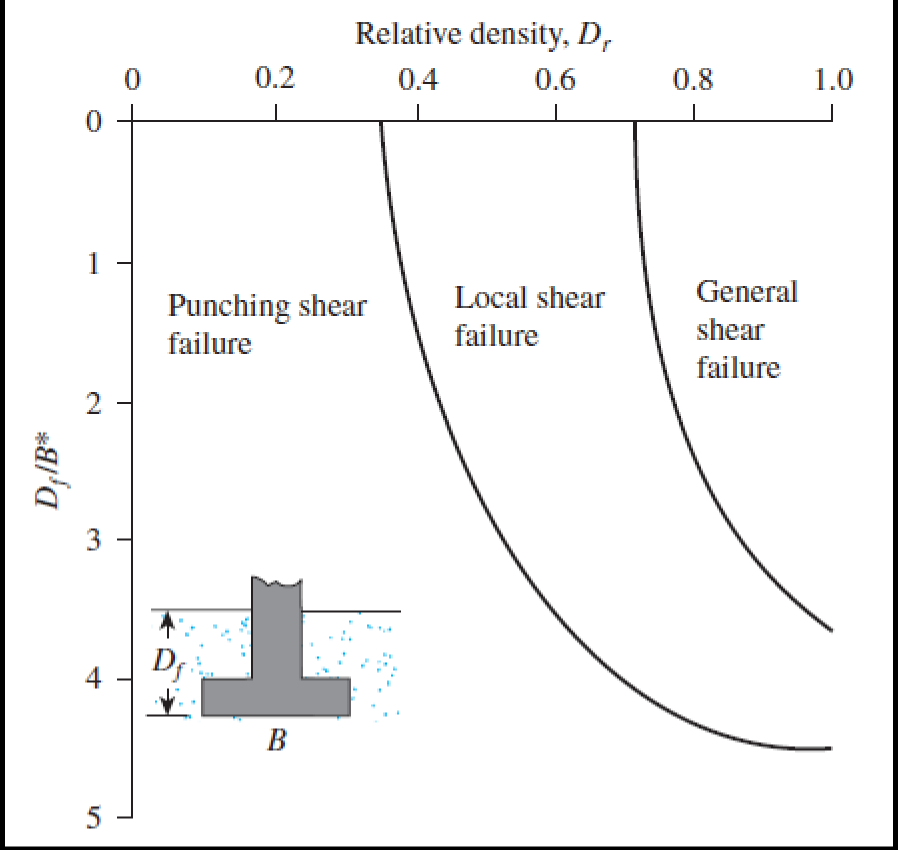


local shear failure;

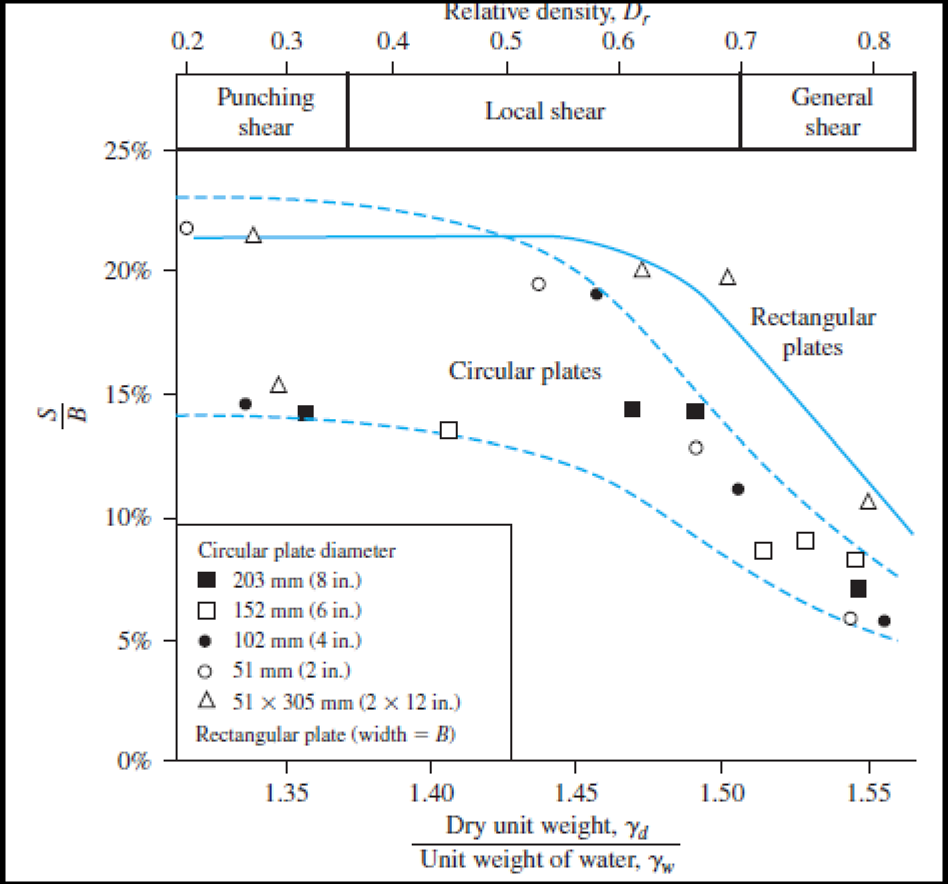


punching shear failure

On the basis of experimental results, Vesic (1973) proposed a relationship for the mode of bearing capacity failure of foundations resting on sands



Modes of foundation failure in sand (After Vesic, 1973)



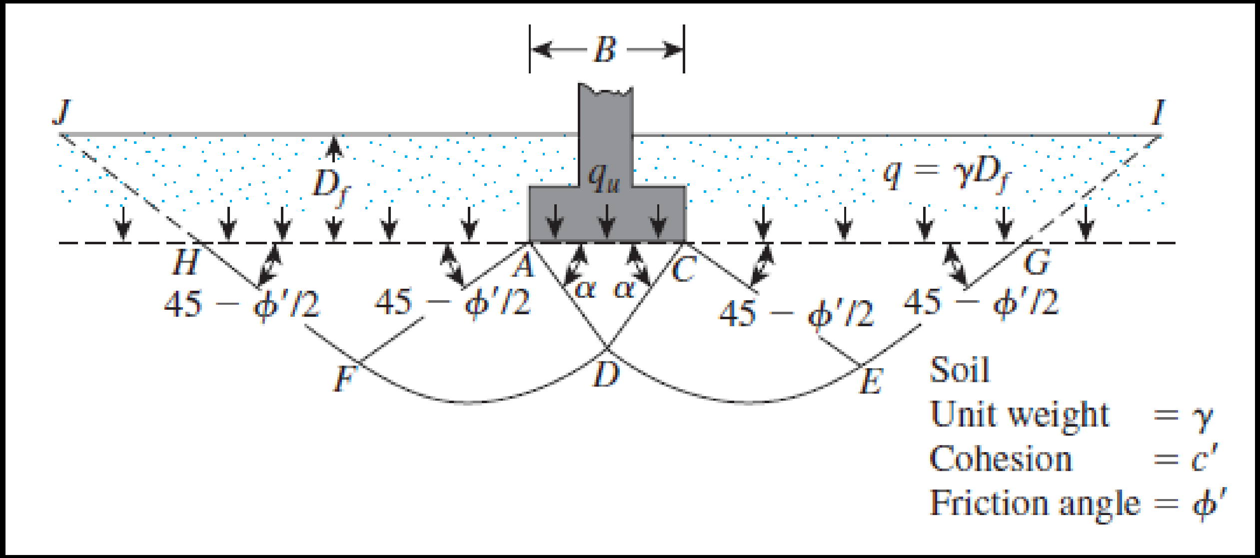
Range of settlement of circular and rectangular plates at ultimate load in sand (Modified from Vesic, 1963)

Terzaghi's Bearing Capacity Theory

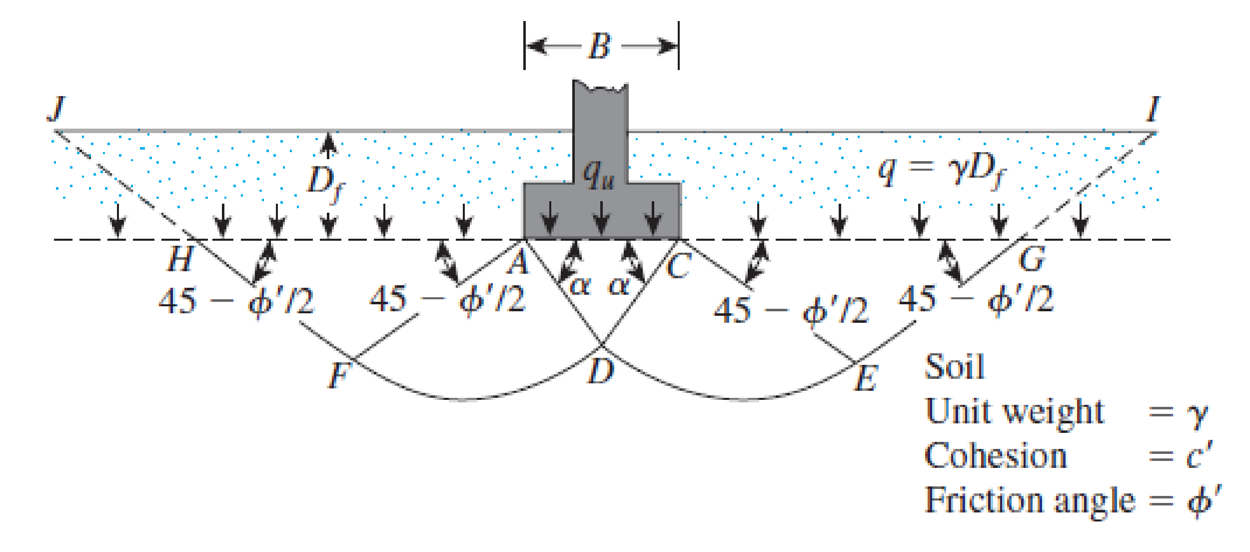
Terzaghi (1943) was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations

Foundations with equal to 3 to 4 times their width may be defined as *shallow foundations*.

Terzaghi suggested that for a *continuous,* or *strip, foundation*the failure surface in soil at ultimate load may be assumed to be similar to that shown in Figure



Bearing capacity failure in soil under a rough rigid continuous (strip) foundation



The effect of soil above the bottom of the foundation may also be assumed to be replaced by an equivalent surcharge.

The failure zone under the foundation can be separated into three parts

1. The *triangular zone ACD* immediately under the foundation
2. The *radial shear zones ADF* and *CDE,* with the curves *DE* and *DF* being arcs of a logarithmic spiral
3. Two triangular *Rankine passive zones AFH* and *CEG*

Using equilibrium analysis, Terzaghi expressed the ultimate bearing capacity in the form

$$q_u = c'N_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma \qquad \text{(continuous or strip foundation)}$$

Where,
c'=cohesion of soil
γ=unit weight of soil
q=γDf
Nc, Nq, Nγ=bearing capacity factors.

$$N_c = \cot\phi'\left[\frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(\frac{\pi}{4}+\frac{\phi'}{2}\right)} - 1\right] = \cot\phi'(N_q - 1)$$

$$N_q = \frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(45+\frac{\phi'}{2}\right)}$$

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)\tan\phi'$$

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma B N_\gamma \quad \text{(square foundation)}$$

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma B N_\gamma \quad \text{(circular foundation)}$$

Table 3.1 Terzaghi's Bearing Capacity Factors—Eqs. (3.4), (3.5), and (3.6) a From Kumbhojkar (1993)

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

Factor of Safety

Calculating the gross *allowable load-bearing capacity* of shallow foundations requires the application of a factor of safety (FS) to the gross ultimate bearing capacity, or

$$q_{\text{all}} = \frac{q_u}{\text{FS}}$$

However, some practicing engineers prefer to use a factor of safety such that

$$\text{Net stress increase on soil} = \frac{\text{net ultimate bearing capacity}}{\text{FS}}$$

The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level

$$q_{\text{net}(u)} = q_u - q$$

$$q_{\text{all(net)}} = \frac{q_u - q}{\text{FS}}$$

Modification of Bearing Capacity Equations for Water Table

Case I. If the water table is located so that 0 <=D1<=Df the factor q in the bearing capacity equations takes the form

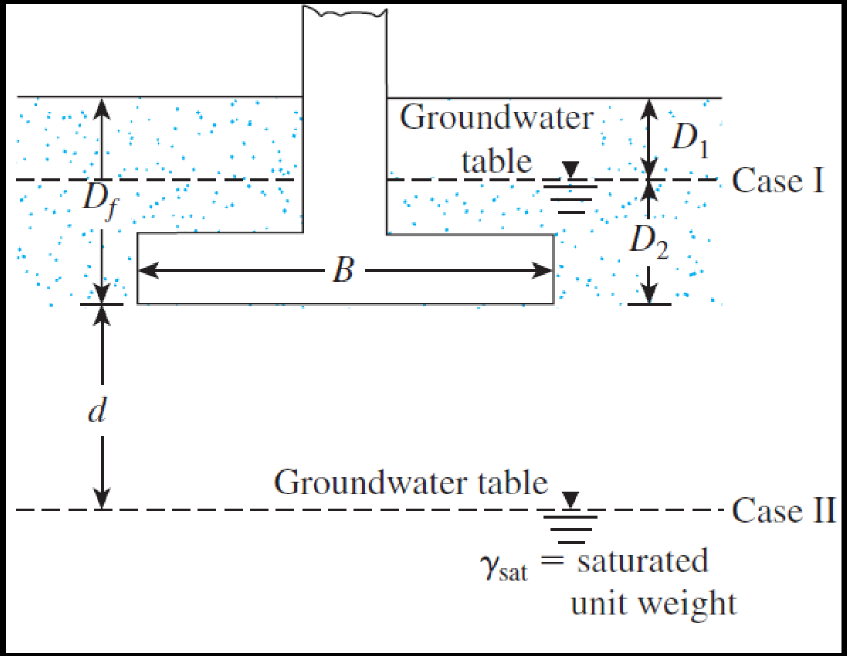
$$q = \text{effective surcharge} = D_1\gamma + D_2(\gamma_{sat} - \gamma_w)$$

Case II. For a water table located so that 0<=D1<=Df

$$q = \gamma D_f$$

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

Case III. When the water table is located so that d>=B, the water will have no effect on the ultimate bearing capacity.



The General Bearing Capacity Equation

The ultimate bearing capacity equations are for continuous, square, and circular foundations only; they do not address the case of rectangular foundations

Also, the equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation

In addition, the load on the foundation may be inclined.

To account for all these shortcomings, Meyerhof (1963) suggested the following form of the general bearing capacity equation:

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

c' = cohesion

q = effective stress at the level of the bottom of the foundation

γ = unit weight of soil

B = width of foundation (= diameter for a circular foundation)

$F_{cs}, F_{qs}, F_{\gamma s}$ = shape factors

$F_{cd}, F_{qd}, F_{\gamma d}$ = depth factors

$F_{ci}, F_{qi}, F_{\gamma i}$ = load inclination factors

N_c, N_q, N_γ = bearing capacity factors

Bearing Capacity Factors

$$N_q = \tan^2\left(45 + \frac{\phi'}{2}\right)e^{\pi \tan \phi'}$$

$$N_c = (N_q - 1)\cot \phi'$$

$$N_\gamma = 2\,(N_q + 1)\tan \phi'$$

Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Shape Factors

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

DeBeer (1970)

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

Inclination Factors

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$$

β = inclination of the load on the foundation with respect to the vertical

Depth Factors

$$\frac{D_f}{B} \le 1$$

For $\phi = 0$:

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

For $\phi' > 0$:

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1$$

$$\frac{D_f}{B} > 1$$

For $\phi = 0$:

$$F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

For $\phi' > 0$:

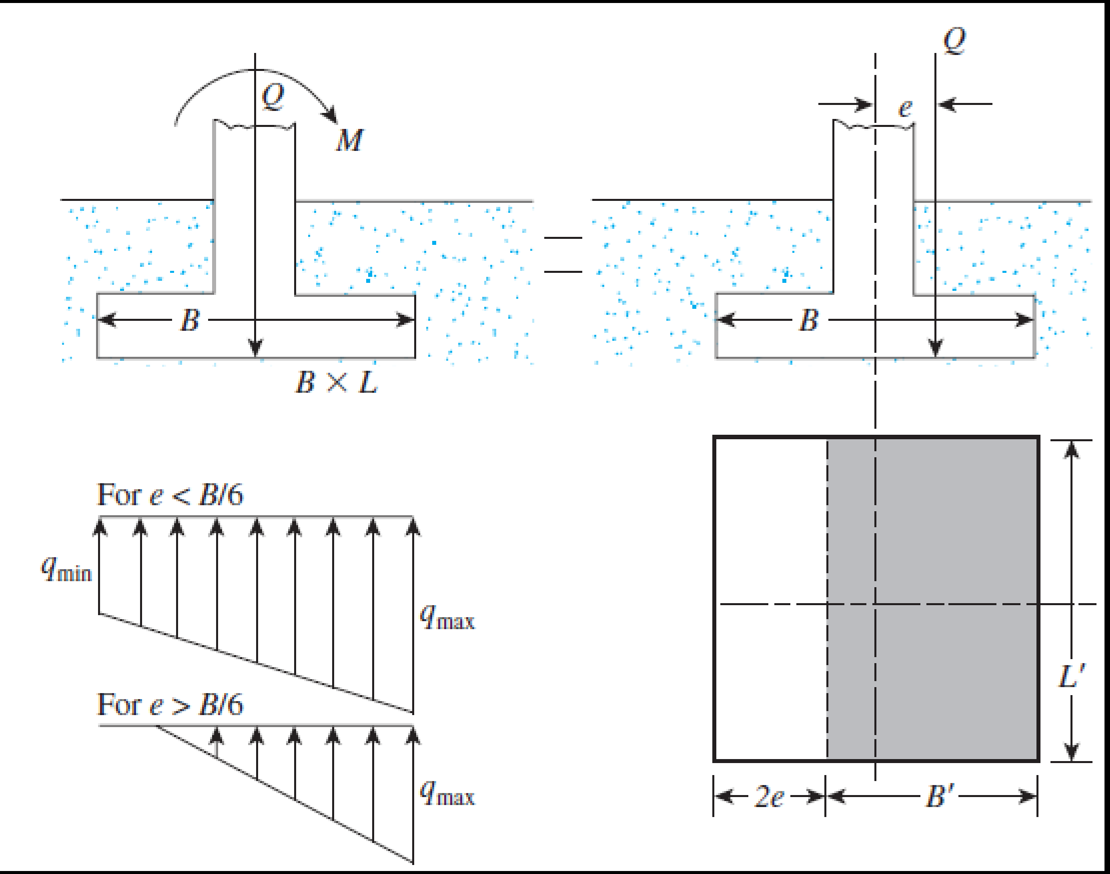
$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$$

$$F_{\gamma d} = 1$$

Eccentrically Loaded Foundations

In several instances, as with the base of a retaining wall, foundations are subjected to moments in addition to the vertical load. In such cases, the distribution of pressure by the foundation on the soil is not uniform.



$$e = \frac{M}{Q}$$

$$q_{max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right)$$

For e>B/6, qmin will be negative, which means that tension will develop. The value of qmax is then.,

$$q_{max} = \frac{4Q}{3L(B - 2e)}$$

Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Effective Area Method (Meyerhoff, 1953)

In 1953, Meyerhof proposed a theory that is generally referred to as the *effective area method*.

Step 1. Determine the effective dimensions of the foundation

B'=effective width=B-2e
L'= effective length=L

Step 2. Use

$$q'_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

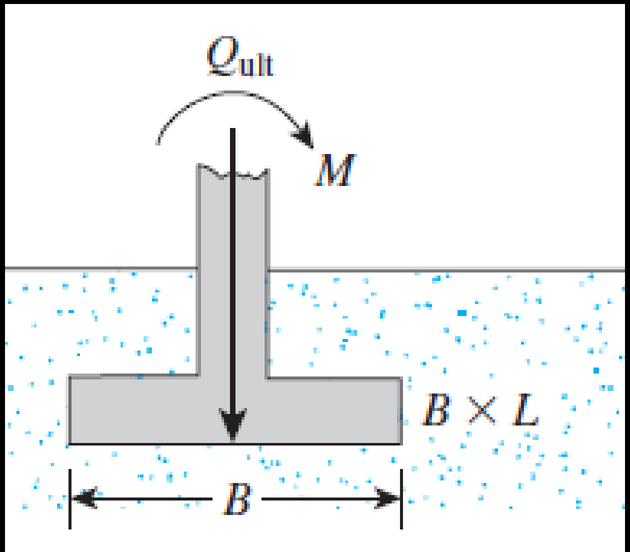
Step 3. The total ultimate load that the foundation can sustain is

$$Q_{ult} = q'_u \overbrace{(B')(L')}^{A'}$$

Step 4. The factor of safety against bearing capacity failure is

$$FS = \frac{Q_{ult}}{Q}$$

Bearing Capacity—Two-way Eccentricity



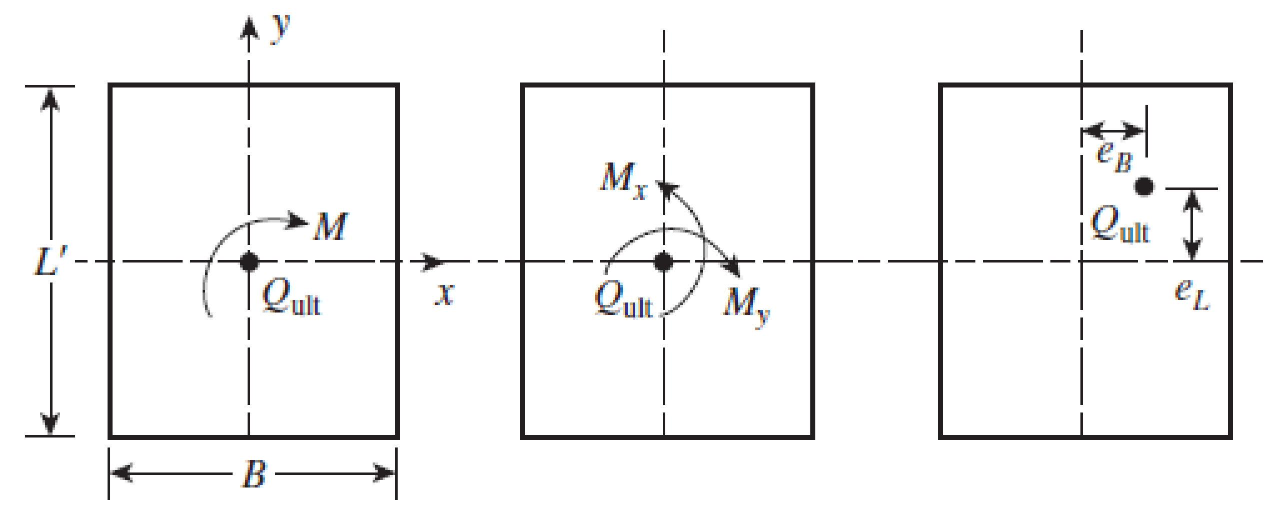
$$e_B = \frac{M_y}{Q_{ult}}$$

$$e_L = \frac{M_x}{Q_{ult}}$$

$$Q_{ult} = q'_u A'$$

$$q'_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$$A' = \text{effective area} = B'L'$$



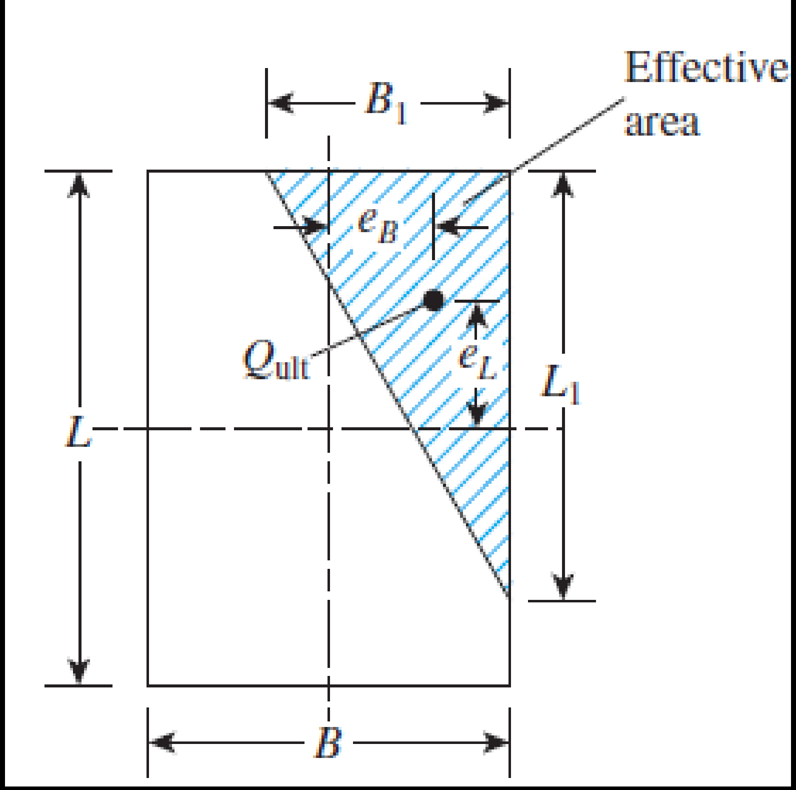
Case I.

$e_L/L \geq \frac{1}{6}$ and $e_B/B \geq \frac{1}{6}$.

$$A' = \tfrac{1}{2}B_1L_1$$

$$B_1 = B\left(1.5 - \frac{3e_B}{B}\right)$$

$$L_1 = L\left(1.5 - \frac{3e_L}{L}\right)$$



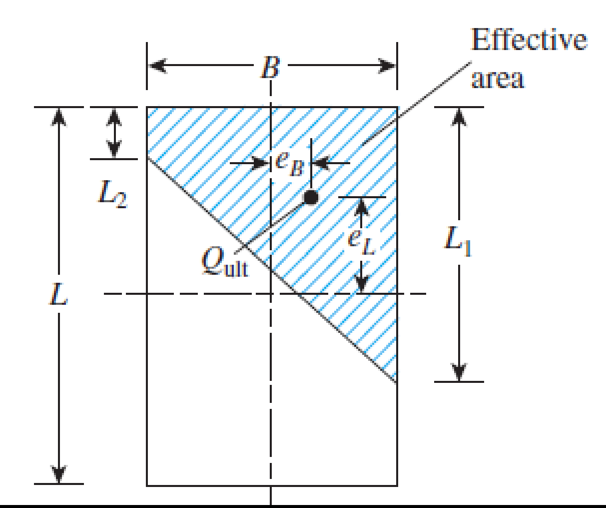
Case II.

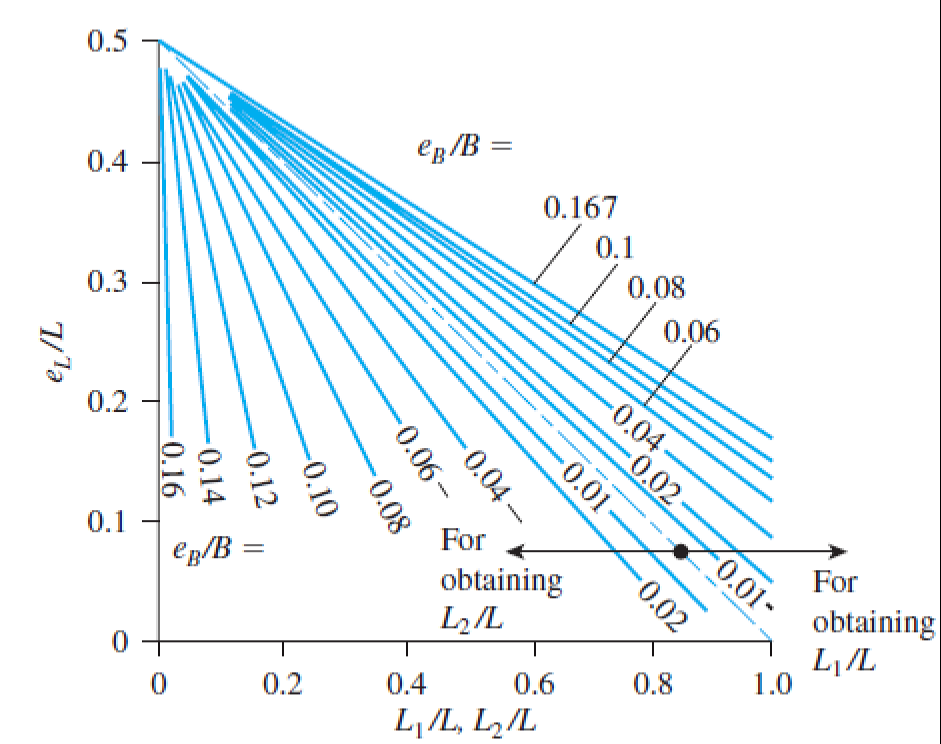
$$A' = \tfrac{1}{2}(L_1 + L_2)B$$

$$e_L/L < 0.5 \text{ and } 0 < e_B/B < \tfrac{1}{6}.$$

$$B' = \frac{A'}{L_1 \text{ or } L_2 \quad \text{(whichever is larger)}}$$

$$L' = L_1 \text{ or } L_2 \quad \text{(whichever is larger)}$$





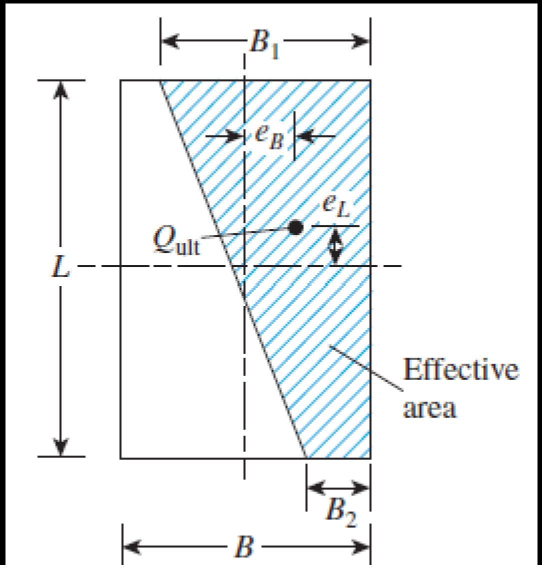
Case III

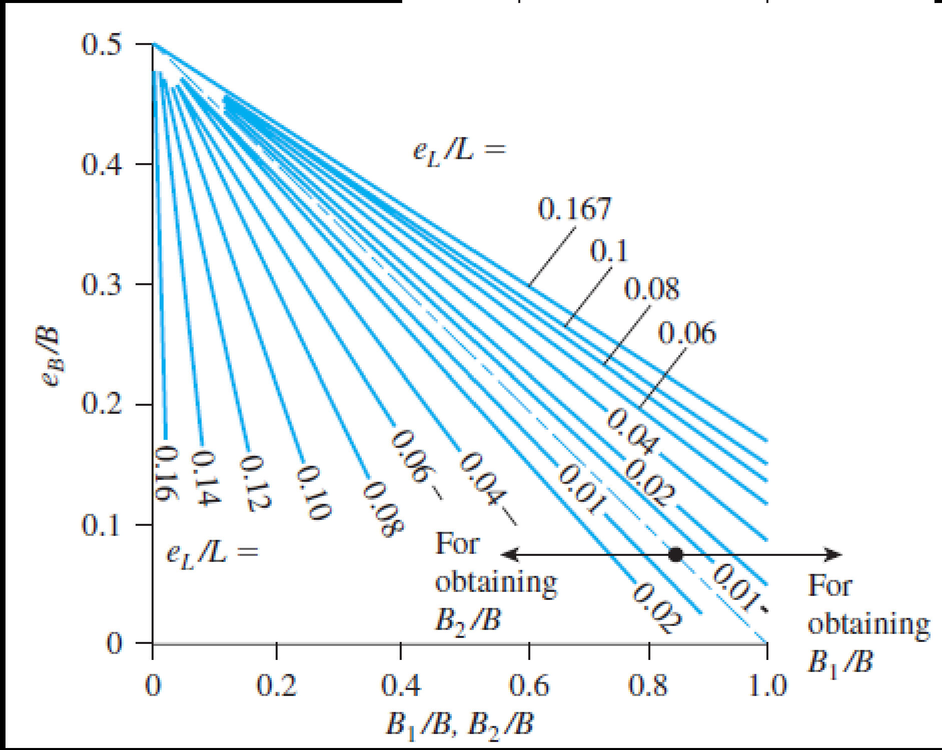
$e_L/L < \frac{1}{6}$ and $0 < e_B/B < 0.5.$

$$A' = \frac{1}{2}(B_1 + B_2)L$$

$$B' = \frac{A'}{L}$$

$$L' = L$$

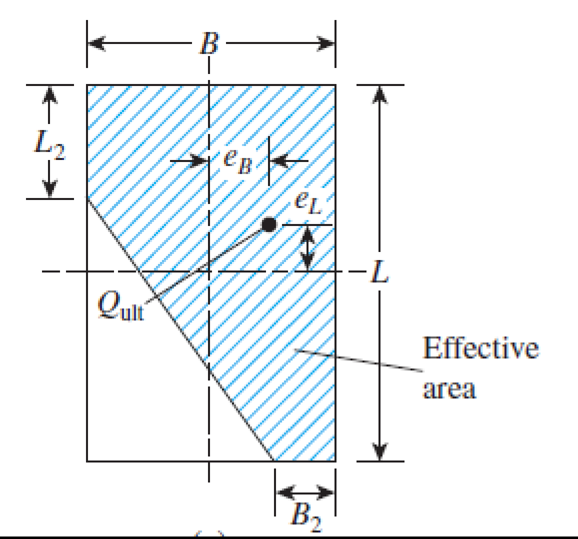


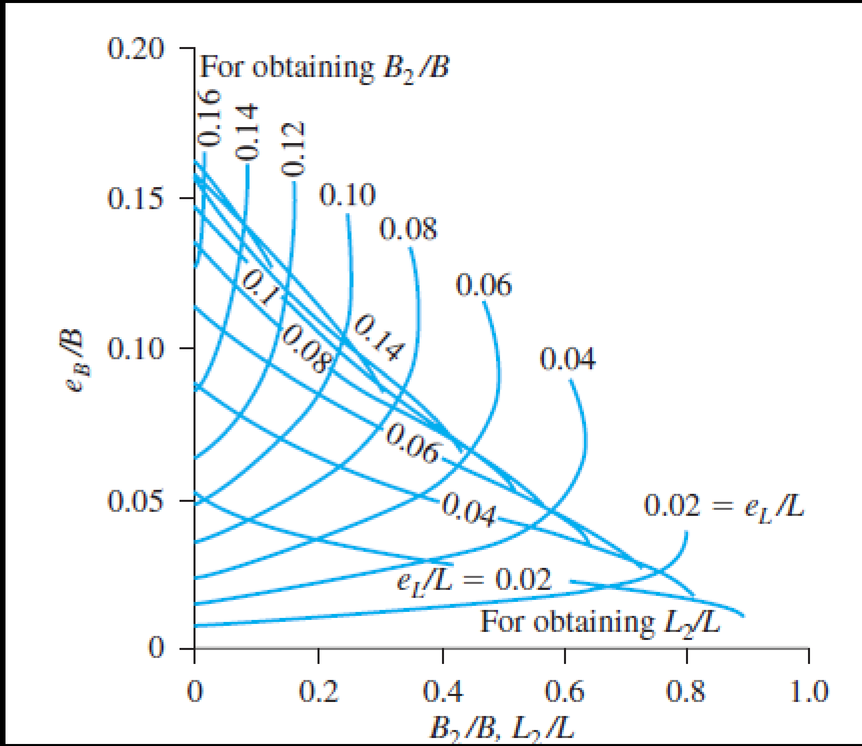


Case IV.

$e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$.

$$A' = L_2 B + \tfrac{1}{2}(B + B_2)(L - L_2)$$





Case V. (Circular Foundation)

$$L' = \frac{A'}{B'}$$

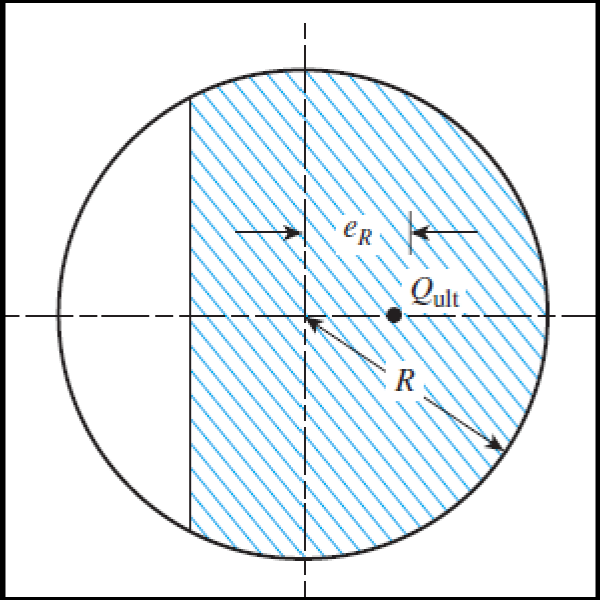


Table 3.8 Variation of A'/R^2 and B'/R with e_R/R for Circular Foundations

e'_R/R	A'/R^2	B'/R
0.1	2.8	1.85
0.2	2.4	1.32
0.3	2.0	1.2
0.4	1.61	0.80
0.5	1.23	0.67
0.6	0.93	0.50
0.7	0.62	0.37
0.8	0.35	0.23
0.9	0.12	0.12
1.0	0	0

REFERENCE

BRAJA DAS, PRONCIPLES OF FOUNDATION ENGINEERING