EXİSTENCE AND UNİQUENESS RESULTS FOR FRACTİONAL DİFFERENTİAL EQUATİONS WİTH DELAY

Okan DUMAN and Faruk DEVELİ
Yildiz Technical University
oduman@yildiz.edu.tr
fdeveli@yildiz.edu.tr

Corresponding Author: oduman@yildiz.edu.tr
(ORCID: https://orcid.org/0000-0001-9848-0759)

ABSTRACT

Fractional differential equations appear in various fields. For examples, it's handled in engineering with physical processes such as thermodynamics, polymer rheology, and mechanics as well as control theory and technical sciences like biophysics. The biggest factor in fractional differentiations being more popular than classical ones lately is that it is effective in explaining real-world problems. But since there is no general technique for obtaining solutions of the dynamic systems specified by fractional calculus, existence and uniqueness theorems have an important place in the literature. In this paper we investigate the existence and uniqueness of solutions for the following fractional order differential equation

$$\begin{cases} {}^{c}D^{\alpha}u(t) = f(t,u(t),u(g(t))) \quad t \in I = [0,T] \\ u(t) = \phi(t) \quad t \in [-h,0] \end{cases}$$

where f $\in C(I \times R^2, R)$, $\Phi \in C([-h,0],R)$, $g \in C(I,[-h,T])$ verifying $g(t) \leq t$ and ${}^{c}D^{\alpha}$ is the fractional derivative of order $\alpha \in (0,1)$ in the sense of Caputo.

Keywords: Fractional order, Delay differential equation, Existence and uniqueness, fixed point.