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THE CRITICAL SPEED OF A MOVING LOAD ON A PRESTRESSED PLATE RESTING ON A PRESTRESSED HALF-PLANE

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Keywords*: critical speed, dynamical response, initial stress, layered half-plane, moving load*

Within the framework of a piecewise homogeneous body model, with the use of the three-dimensional linearized theory of elastic waves in initially stressed bodies, the dynamical response of a system consisting of a prestressed covering layer and a prestressed half-plane to a moving load applied to the free face of the covering layer is investigated. Two types (complete and incomplete) of contact conditions on the interface are considered. The subsonic state is considered, and numerical results for the critical speed of the moving load are presented. The influence of problem parameters on the critical speed is analyzed. In particular, it is established that the prestressing of the covering layer and half-plane increases the critical speed.

1. Introduction

Studies on the dynamical response of layered bodies to moving loads are of great significance in both theoretical and practical sense. The results of these investigations can be employed in many branches of modern industry, such as the design of roadbeds and bridges traversed by high-speed trains, parking garages, aircraft carriers, ballistic systems such as rail guns, high-speed precision machining, magnetic disk drives, and so forth. The corresponding problems were considered, e. g., in [1-5], where the classical linear theory of elastodynamics was used. In these investigations, systems consisting of a packet of layers and a half-space were considered. The motion of layers were described by the Kirchhoff or Timoshenko beam theory, but the motion of the half-space by the linear theory of elastic waves. It should be noted that many modern elastodynamics problems with moving loads cannot be solved within the framework of the classical linear theory of elastic waves, among which are also those for initially stressed layered bodies. Initial stresses occur in layered systems after their manufacture and assembly, and they must be taken into account in studying the dynamical response of such systems to a moving load.

Under certain conditions, the foregoing type of problems can be solved with the use of the three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). The construction of field equations of the theory and their use in investigating the wave propagation problem for initially stressed bodies are detailed in [6]. A review of these studies is given in [7, 8]. In [9-14], time-harmonic stress fields in layered prestressed bodies were examined by employing the TLTEWISB.

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At the same time, only a few studies have been performed until now on the dynamical response of a prestressed layered half-space to a moving load [15, 16]. In [15], the dynamical response of a system consisting of a layer and prestressed half-plane was considered. The motion of the covering layer was described by the Timoshenko beam theory, but the motion of the half plane by the TLTEWISB. The solution to the corresponding boundary-value problem was found by employing Fourier transforms. Numerical investigations were performed in the case where the constitutive relations for the half-plane material were given by a harmonic potential. The speed of the moving load was assumed constant, and the subsonic case was considered. As a result, the influence of problem parameters on the critical speed was clarified.

In [16], the problem considered in [15] was studied by using the complex potentials of the TLTEWISB.

In the present paper, the investigations carried out in [15, 16] are developed for the case where the covering layer has also an initial stress, and the equation of motion of this layer is obtained by using the TLTEWISB. The influence of problem parameters on the critical speed of the moving load is studied. Moreover, the stress distribution on the interface plane is investigated.

2. Problem Formulation

Let us consider a prestressed half-plane covered with a prestressed layer. The points of the layer and half-plane are specified by Lagrangian coordinates in a Cartesian reference frame $Ox_1x_2x_3$ (Fig. 1.). Before assembling, the layer and the half-plane had been stressed separately in the Ox_1-axis direction, so that a uniaxial homogeneous initial stress state exists in each of them. The covering layer and the half-plane occupy the regions $\{ x_1 , h x_2 0, x_3 \}$ and $\{ x_1 , x_2 h, x_3 \}$, respectively (Fig. 1). The Ox_3 axis is perpendicular to the figure plane and therefore is not shown in Fig. 1.

The stresses, strains, and displacements related to the layer and half-plane are denoted by the superscripts (1) and (2), respectively. The quantities related to the initial stresses are denoted by the superscript $(m)0$, where $m = 1, 2$.

The linearly elastic materials of the layer and the half-plane are taken to be homogeneous, isotropic, and moderately rigid. The initial stress states $_{ij}$ in the constituents can be written in the form

$$ {}^{(m)0}_{11} \quad \text{const}_m , \qquad {}^{(m)0}_{ij} \quad 0 \text{ at } ij \quad 11. $$

Since the initial strains are small, the coordinates of the layer and half-plane points in the natural and initial states coincide.

Thus, within the framework of foregoing assumptions, we assume that, on the free upper face of the covering layer, a lineal force moves in the Ox_1-axis direction with a constant speed V (Fig. 1.). It is required to determine the dynamical response of the layered system to the loading. We will consider only the subsonic state, i. e.,

$$ V \quad \min (C_2^{(1)}, C_2^{(2)}), \quad C_2^{(m)} \quad \sqrt{ {}^{(m)} / {}^{(m)}}, \quad m = 1, 2, $$

where $C_2^{(m)}$, $^{(m)}$, and $^{(m)}$ are the speed of distortion waves and the shear modulus and density of constituent materials.

According to [6], the complete system of equations of the TLTEWISB in the case of small initial strains $_{ij}$ is

$$ \frac{{}^{(m)}_{ij}}{x_i} \quad {}^{(m),0}_{11} \frac{{}^2 u_j^{(m)}}{x_1^2} \quad {}^{(m)} \frac{{}^2 u_j^{(m)}}{t^2}, \tag{1} $$

$$ {}^{(m)}_{ij} \quad {}^{(m)} \; {}^{(m)} \; {}^{j}_{i} \quad 2 \; {}^{(m)} \; {}^{(m)}_{ij}, \tag{2} $$

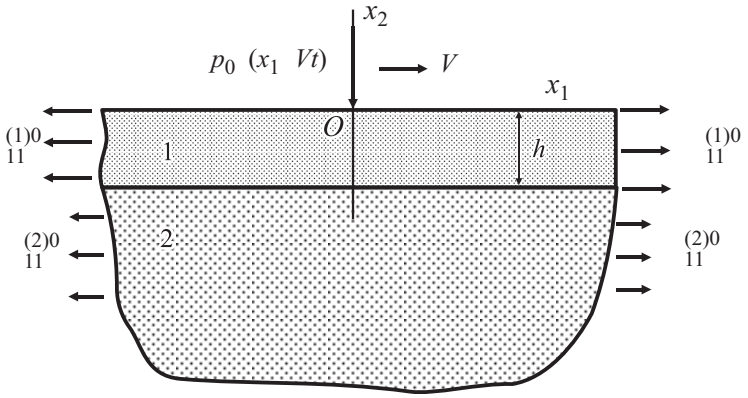


Fig. 1.Geometry of a prestressed half-plane covered with a prestresses layer: 1 — layer and 2 — half-plane.

$$\begin{matrix}(m)\\ \end{matrix}\quad \begin{matrix}(m)\\ 11\end{matrix}\quad \begin{matrix}(m)\\ 22\end{matrix}, \quad \begin{matrix}(m)\\ ij\end{matrix}\quad \frac{1}{2}\ \frac{u_i^{(m)}}{x_j}\quad \frac{u_j^{(m)}}{x_i}\ , \tag{3}$$

where $i, j, m = 1, 2$; $^{(m)}$ and $^{(m)}$ are also called Lamé constants.

Note that Eqs. (1)-(3) are satisfied within the covering layer and the half-plane separately. According to the above-mentioned, on the upper face of the covering layer, the following boundary conditions must be satisfied:

$$\begin{matrix}(1)\\ 12\end{matrix}\Big|_{x_2\ \ 0}\quad 0, \qquad \begin{matrix}(1)\\ 22\end{matrix}\Big|_{x_2\ \ 0}\quad P\ (x_1\ \ Vt), \tag{4}$$

where (x) is the Dirac delta function.

As in [15], we will consider two types (complete and incomplete) of contact conditions on the interface plane between the covering layer and half-plane. For the complete contact conditions, we assume

$$\begin{matrix}(1)\\ i2\end{matrix}\Big|_{x_2\quad h}\quad \begin{matrix}(2)\\ i2\end{matrix}\Big|_{x_2\quad h}, \quad u_i^{(1)}\Big|_{x_2\quad h}\quad u_i^{(2)}\Big|_{x_2\quad h}, \quad i = 1, 2, \tag{5}$$

and for the incomplete ones

$$\begin{matrix}(1)\\ 12\end{matrix}\Big|_{x_2\quad h}\quad 0, \quad \begin{matrix}(2)\\ 12\end{matrix}\Big|_{x_2\quad h}\quad 0, \ u_2^{(1)}\Big|_{x_2\quad h}\quad u_2^{(2)}\Big|_{x_2\quad h}, \quad \begin{matrix}(1)\\ 22\end{matrix}\Big|_{x_2\quad h}\quad \begin{matrix}(2)\\ 22\end{matrix}\Big|_{x_2\quad h}. \tag{6}$$

In addition, we also assume that as

$$|u_i^{(2)}|, |\ \begin{matrix}(2)\\ ij\end{matrix}|\quad 0 \ \text{ at } \ x_2\qquad . \tag{7}$$

This terminates the formulation of the problem. Thus, the investigation of the dynamical response of the layered system to a moving load is reduced to the solution of boundary-value and contact problem (1)-(7).

3. Solution Method

Form Eqs. (1)-(3), we obtain the following equations of motion in terms of perturbations of displacements u_i:

$$
\begin{aligned}
&\left({}^{(m)} \quad {}^{(m)} \quad {}^{(m)0}_{11} \right) \frac{{}^{2}u_1^{(m)}}{x_1^2} \quad {}^{(m)} \frac{{}^{2}u_1^{(m)}}{x_2^2} \\
&\left({}^{(m)} \quad {}^{(m)} \right) \frac{{}^{2}u_2^{(m)}}{x_1 \; x_2} \quad {}^{(m)} \frac{{}^{2}u_1^{(m)}}{t^2}, \\
&\left({}^{(m)} \quad {}^{(m)} \right) \frac{{}^{2}u_1^{(m)}}{x_1 \; x_2} \quad \left({}^{(m)} \quad {}^{(m)0}_{11} \right) \frac{{}^{2}u_1^{(m)}}{x_1^2} \\
&\left({}^{(m)} \quad 2 \; {}^{(m)} \right) \frac{{}^{2}u_2^{(m)}}{x_2^2} \quad {}^{(m)} \frac{{}^{2}u_2^{(m)}}{t^2}.
\end{aligned}
\tag{8}
$$

In the coordinate system

$$
x_2 \quad x_2, \quad x_1 \quad x_1 \quad Vt, \tag{9}
$$

which moves together with the loading force, Eqs. (8) have the form

$$
\begin{aligned}
&\frac{{}^{(m)}}{{}^{(m)}} \quad 2 \quad \frac{{}^{(m)0}_{11}}{{}^{(m)}} \quad \frac{V^2}{(C_2^{(m)})^2} \quad \frac{{}^{2}u_1^{(m)}}{x_1^2} \quad \frac{{}^{2}u_1^{(m)}}{x_2^2} \quad \frac{{}^{(m)}}{{}^{(m)}} \quad 1 \quad \frac{{}^{2}u_2^{(m)}}{x_1 \; x_2} \quad 0, \\
&\frac{{}^{(m)}}{{}^{(m)}} \quad 1 \quad \frac{{}^{2}u_1^{(m)}}{x_1 \; x_2} \quad 1 \quad \frac{{}^{(m)0}_{11}}{{}^{(m)}} \quad \frac{V^2}{(C_2^{(m)})^2} \quad \frac{{}^{2}u_2^{(m)}}{x_1^2} \quad \frac{{}^{(m)}}{{}^{(m)}} \quad 2 \quad \frac{{}^{2}u_2^{(m)}}{x_2^2} \quad 0,
\end{aligned}
\tag{10}
$$

where the primes of x_1 and x_2 are omitted. In this case, the second boundary condition in (4) is replaced by

$$
{}^{(1)}_{22}\Big|_{x_2 \; 0} \quad P \; (x_1). \tag{11}
$$

The other conditions in (4)-(7) remain valid in coordinates (9).

In order to solve equation system (10), we apply the Fourier transform

$$
f_F \; (s, x_2) \qquad f(x_1, x_2) e^{\; isx_1} dx_1
$$

to it. As a result, we have

$$
\begin{aligned}
&s^2 a^{(m)} u_{1F}^{(m)} \quad \frac{d^2 u_{1F}^{(m)}}{dx_2^2} \quad isb^{(m)} \frac{du_{2F}^{(m)}}{dx_2} \quad 0, \\
&isb^{(m)} \frac{du_{1F}^{(m)}}{dx_2} \quad s^2 c^{(m)} u_{2F}^{(m)} \quad (b^{(m)} \quad 1) \frac{d^2 u_{2F}^{(m)}}{dx_2^2} \quad 0,
\end{aligned}
\tag{12}
$$

where

$$a^{(m)} \quad \frac{(m)}{(m)} \quad 2 \quad \frac{(m)0}{11} \quad \frac{V^2}{(c_2^{(m)})^2},$$

$$b^{(m)} \quad \frac{(m)}{(m)} \quad 1, \qquad c^{(m)} \quad 1 \quad \frac{(m)0}{11} \quad \frac{V^2}{(c_2^{(m)})^2}.$$

After some mathematical manipulations, we obtain from (12) the equations

$$(b^{(m)} \quad 1)\frac{d^4 u_{2F}^{(m)}}{dx_2^4} \quad [s^2 (b^{(m)})^2 \quad s^2 c^{(m)} \quad s^2 a^{(m)} (b^{(m)} \quad 1)]\frac{d^2 u_{2F}^{(m)}}{dx_2^2}$$

$$s^4 a^{(m)} c^{(m)} u_{2F}^{(m)} \quad 0, \tag{13}$$

$$\frac{du_{1F}^{(m)}}{dx_2} \quad is\frac{c^{(m)}}{b^{(m)}} u_{2F}^{(m)} \quad \frac{i(b^{(m)} \quad 1)}{sb^{(m)}} \frac{d^2 u_{2F}^{(m)}}{dx_2^2}.$$

From Eq. (13) it follows

$$u_{2F}^{(1)} \quad F_1^{(1)}(s)e^{K_1^{(1)} x_2} \quad F_2^{(1)}(s)e^{\ K_1^{(1)} x_2} \quad F_3^{(1)}(s)e^{K_2^{(1)} x_2} \quad F_4^{(1)}(s)e^{\ K_2^{(1)} x_2},$$

$$u_{2F}^{(2)} \quad F_2^{(2)}(s)e^{\ K_1^{(2)} x_2} \quad F_4^{(2)}(s)e^{\ K_2^{(2)} x_2}, \tag{14}$$

where

$$K_1^{(m)} \quad \sqrt{\frac{A^{(m)}}{2} \quad \sqrt{\frac{A^{(m)}}{2}^2 \quad B^{(m)}}}, \quad K_2^{(m)} \quad \sqrt{\frac{A^{(m)}}{2} \quad \sqrt{\frac{A^{(m)}}{2}^2 \quad B^{(m)}}},$$

$$A^{(m)} \quad \frac{s^2 b^{(m)} \quad s^2 c^{(m)} \quad s^2 a^{(m)} (b^{(m)} \quad 1)}{b^{(m)} \quad 1}, \quad B^{(m)} \quad \frac{s^4 a^{(m)} c^{(m)}}{b^{(m)} \quad 1}.$$

Thus, from Eqs. (14), (2) and (3), the Fourier transforms of all sought-for quantities are found. The unknowns $F_1^{(1)}(s)$, ..., $F_4^{(1)}(s)$, $F_2^{(2)}(s)$, and $F_4^{(2)}(s)$ appearing in these transforms are obtained from boundary condition (11) and contact condition (5) (for a complete contact) or (6) (for an incomplete contact), which form a closed system of algebraic equations. Note that condition (7) was already used for determining $u_{2F}^{(2)}$ in (14).

From the algebraic equations, we find the aforementioned unknowns and then employ the inverse transform

$$f(x_1, x_2) \quad \frac{1}{2} \quad f_F(s, x_2) e^{isx_1} ds. \tag{15}$$

As follows from the problem statement, integral (15) can be reduced to the following ones:

$$f(x_1, x_2) \quad \frac{1}{} \quad _0 f_F(s, x_2)\cos(sx_1) ds \ \text{ for } \ u_{2F}^{(m)}, \ {}_{22}^{(m)}, \ {}_{11}^{(m)}, \ {}_{22}^{(m)}, \text{ and } \ {}_{11}^{(m)},$$

$$f(x_1, x_2) \quad \frac{1}{} \quad f_F(s, x_2)\sin(sx_1)\,ds \text{ for } u_{1F}^{(m)}, \ {}_{12}^{(m)}, \text{ and } {}_{12}^{(m)}.$$

In numerical calculations, these improper integrals must be replaced with the corresponding proper integrals by using the relation

$$\int_0^{} (.)\,ds \quad \int_0^{S} (.)\,ds. \tag{16}$$

The values of S in (16) are determined from convergence criteria for the improper integrals.

Numerical investigations show that, in general, at fixed values of problem parameters, for any value of the speed V, the functions $u_{iF}^{(m)}$ and ${}_{ijF}^{(m)}$ have singular points with respect to sh. Let us determine these points. The unknowns $F_1^{(1)}(s)$, ..., $F_4^{(1)}(s)$, $F_2^{(2)}(s)$, and $F_4^{(2)}(s)$ in (14) can be expressed from the aforementioned system of algebraic equations in the form

$$F_1^{(1)}(s), \ldots, F_4^{(1)}(s), F_2^{(2)}(s), F_4^{(2)}(s) \quad \frac{1}{\det\| \ {}_{nm}(s)\|} \det\left\| {}_{nm}^{F_1^{(1)}(s)}(s)\right\|, \ldots,$$

$$\det\left\| {}_{nm}^{F_4^{(1)}(s)}(s)\right\|, \det\left\| {}_{nm}^{F_2^{(2)}(s)}(s)\right\|, \det\left\| {}_{nm}^{F_4^{(2)}(s)}(s)\right\| .$$

The singular points coincide with roots of the equation in s

$$\det\| \ {}_{nm}(s)\| \quad 0; \quad n, m = 1, 2, \ldots, 6, \tag{17}$$

where ${}_{nm}(s)$ are the coefficients of unknowns in the system. Note that the expressions for

$$\det\left\| {}_{nm}^{F_1^{(1)}(s)}(s)\right\|, \ldots, \det\left\| {}_{nm}^{F_4^{(1)}(s)}(s)\right\|, \ \det\left\| {}_{nm}^{F_2^{(2)}(s)}(s)\right\|, \text{ and } \det\left\| {}_{nm}^{F_4^{(2)}(s)}(s)\right\|$$

are obtained from $\| \ {}_{nm}(s)\|$ by replacing the corresponding column of $\| \ {}_{nm}(s)\|$ with the right side of the equation system.

The order of singularity (denoted by r) of the integrated values coincide with the order of roots of Eq. (17). It is known that, if $0 \quad r \quad 1$, integral (16) can be calculated by using a well-known algorithm. For $r = 1$, the calculation of the integral is performed in the sense of Cauchy principal value. In the case $r \quad 1$, the integral does not have any meaning, and the speed corresponding to this case is called the "critical speed". At the critical speed, a resonance-type phenomenon takes place.

It is obvious that the critical speed corresponds to the local minimum (or maximum) of the function $V \quad V(sh)$ which satisfies Eq. (17). The main problem for layered materials under a moving load in the subsonic state is the determination of the critical speed (denoted by V_{cr}) and investigation of the influence of problem parameters on its values.

The other problem is the determination of stress–strain state in the materials at $V \quad V_{\text{cr}}$. In this case, integral (16) is calculated by using the algorithm developed in [9-14].

Now, we will consider the numerical results pertinent to the problems.

4. Numerical Results and Discussion

Let us introduce the designations

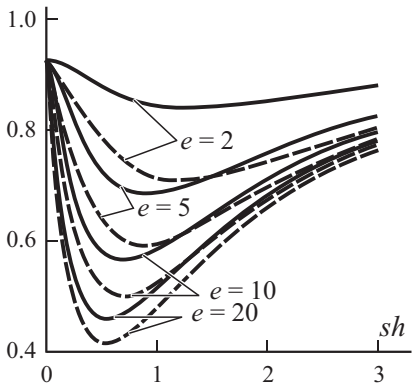


Fig. 2. Graphs of the relationship (sh) for complete (——) and incomplete (- - -) contact conditions at $_1$ $_2 = 0$.

TABLE 1. Critical Speed $_{cr}$ for Various Values of $_1$ and e under Complete (Numerator) and Incomplete (Denominator) Contact Conditions at $_2 = 0$ and $_{12} = 1$

e	$_1$			
	0	0.005	0.01	0.03
2	0.8414 / 0.7093	0.8468 / 0.7154	0.8521 / 0.7214	0.8723 / 0.7442
10	0.5664 / 0.5016	0.5764 / 0.5126	0.5863 / 0.5233	0.6240 / 0.5640

TABLE 2. Critical Speed $_{cr}$ for Various Values of $_2$ and e under Complete (Numerator) and Incomplete (Denominator) Contact Conditions at $_1 = 0$ and $_{12} = 1$

e	$_2$			
	0	0.005	0.01	0.03
2	0.8414 / 0.7093	0.8437 / 0.7123	0.8454 / 0.7153	0.8540 / 0.7266
10	0.5664 / 0.5016	0.5677 / 0.5034	0.5690 / 0.5052	0.5741 / 0.5121

$$\frac{V}{c_2^{(1)}}, \qquad \frac{c_2^{(1)}}{c_2^{(2)}}, e \quad \frac{E^{(1)}}{E^{(2)}}, \quad _1 \quad \frac{^{(1)0}_{11}}{E^{(1)}}, \text{ and } \quad _2 \quad \frac{^{(2)0}_{11}}{E^{(2)}},$$

and assume that $^{(1)}$ $^{(2)} = 0.3$, where $E^{(m)}$ and $^{(m)}$ are Young's modulus and the Poisson ratio of an mth material. First, we consider the case where $_{12} = 1$ and $_1$ $_2 = 0$ and analyze graphs of the functions (sh) found by solving Eq. (17). The numerical results obtained (not given here) show that the functions have minima only in the cases where e 1. Graphs of (sh) for various values of e are shown in Fig. 2. It is seen that $_{cr}$ decreases monotonically with growing e.

The numerical results given in Tables 1, 2, and 3 show that $_{cr}$ increases monotonically with initial tensile stresses in the covering layer and half-plane, but the initial compression of the half-plane lowers the values of $_{cr}$.

TABLE 3. Critical Speed $_{cr}$ for Various Values of $_2$ and e under Initial Compression for Complete (Numerator) and Incomplete (Denominator) Contact Conditions at $_1 = 0$ and $_{12} = 1$

e	$_2$			
	0	–0.005	–0.01	–0.03
2	0.8414 / 0.7093	0.8391 / 0.7062	0.8367 / 0.7031	0.8263 / 0.6898
10	0.5664 / 0.5016	0.5650 / 0.4998	0.5637 / 0.4979	0.5580 / 0.4900

TABLE 4. Critical Speed $_{cr}$ under Complete and Incomplete Contact Conditions for $_{12}$ $\sqrt{0.5e}$ at $_1$ $_2 = 0$*

e	Complete contact	Incomplete contact
2	0.841 / 0.839	0.709 / 0.719
10	0.428 / 0.427	0.373 / 0.372

*Numerator — results of the present investigation; denominator — data from [15].

TABLE 5. Critical Speed $_{cr}$ for Various Values of $_2$ and e under Complete (Numerator) and Incomplete (Denominator) Contact Conditions for $_{12}$ $\sqrt{0.5e}$ at $_1 = 0$

e	$_2$			
	0	0.005	0.01	0.03
2	0.8414 / 0.7093	0.8437 / 0.7123	0.8454 / 0.7153	0.8541 / 0.7266
10	0.4284 / 0.3730	0.4307 / 0.3754	0.4329 / 0.3777	0.4415 / 0.3868

TABLE 6. Critical Speed $_{cr}$ for Various Values of $_1$ and e under Complete (Numerator) and Incomplete (Denominator) Contact Conditions for $_{12}$ $\sqrt{0.5e}$ and $_2 = 0$

e	$_1$			
	0	0.005	0.01	0.03
2	0.8414 / 0.7093	0.8468 / 0.7154	0.8521 / 0.7214	0.8723 / 0.7442
10	0.4284 / 0.3730	0.4319 / 0.3784	0.4348 / 0.3833	0.4358 / 0.3995

Let us consider now the case where $^{(2)} / ^{(1)} = 0.5$, i. e., $_{12}$ $\sqrt{0.5e}$. In Table 4, the values of critical speed $_{cr}$ obtained in the present investigation for various $_{cr}$ at $_1$ $_2 = 0$ are compared with corresponding results found in [15], and a good agreement is seen to exist between then.

Tables 5, 6, and 7 show the influence of initial stresses, i. e., of the parameters $_1$ and $_2$, on the values of $_{cr}$. It follows from these tables that, in the qualitative sense, the effect of $_1$ and $_2$ on $_{cr}$ is the same as in the case $_{12} = 1$.

TABLE 7. Critical Speed $_{cr}$ for Various Values of $_2$ and e under Initial Compression for Complete (Numerator) and Incomplete (Denominator) Contact Conditions with $_{12}$ $\sqrt{0.5e}$ at $_1 = 0$

e	$_2$			
	0	0.005	0.01	0.03
2	0.8414 / 0.7093	0.8391 / 0.7062	0.8367 / 0.7031	0.8263 / 0.6898
10	0.4284 / 0.3730	0.4261 / 0.3706	0.4238 / 0.3682	0.4141 / 0.3582

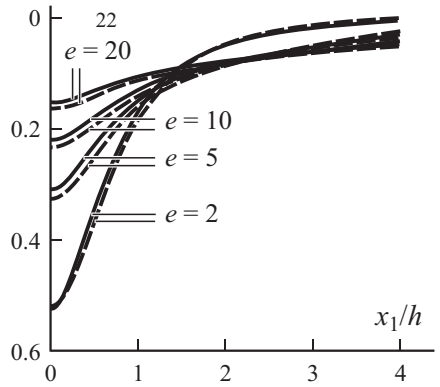


Fig. 3. The normal stress $_{22}$ under complete (——) and incomplete (- - -) contact conditions vs. x_1/h for various values of e at $= 0.3$ and $_1 = _2 = 0$.

The magnitude of the stress $_{22}(x_1)$ $^{(1)}_{22}(x_1, h)$ $^{(2)}_{22}(x_1, h)$ increases monotonically with on the interval $(0, _{cr})$, and $|_{22}|$ as $_{cr}$. The absolute maximum of $_{22}(x_1)$ is reached at $x_1/h = 0$.

The graphs of $_{22}(x_1/h)$ in the case $_{12}$ $\sqrt{0.5e}$ are given in Fig. 3. It is seen that $|_{22}|$ decreases with e. In this case, the values of $|_{22}|$ obtained for the incomplete contact conditions are greater than those for the complete ones between the covering layer and the half-plane, and the influence of the type of contact conditions on $_{22}$ increases with e.

5. Conclusions

In the present paper, within the framework of a piecewise-homogenous body model, with the use of TLTEWISB, the dynamical response of a system consisting of a prestressed covering layer and a prestressed half-plane to a moving load has been investigated. An algorithm for determining the critical speed of the moving load was developed.

The numerical results obtained allow us to draw the following conclusions:

a critical speed arises only in the cases where Young's modulus of the covering layer $E^{(1)}$ is greater than that of the underlaying half-plane $E^{(2)}$;

the critical speed decreases with growing ratio $E^{(1)}/E^{(2)}$;

the prestressing the covering layer and the half-plane increases the critical speed;

the critical speed decreases with growing initial compression of the half-plane;

the critical speed at complete contact conditions is greater than that at incomplete ones.

These results are also significant in the linear theory of elastodynamics in the absence of initial stresses in the components of the layered system considered.

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