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Determination of the actual deflection of the cutting tool ın turning by the Castigliano theorem

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ABSTRACT

The turning method is a widely utilized technique in machining. The effect of machining parameters on the machining quality in turning is quite high. A longer tool overhang reduces tool rigidity, leading to increased tool deflection and subsequently, a decrease in machining quality. The tool overhang should be as short as possible in order to reduce tool deflection and eliminate the negative effects of vibration. Another critical factor affecting machining quality, as much as the tool overhang length is the distance between the main cutting edge and the tool holder axis. In the turning process, the alteration of the main cutting-edge position significantly influences the amount of tool deflection. When the main cutting edge is positioned farther from the tool holder axis, it generates a greater torsional moment effect, which negatively impacts machining quality. However, although the main cutting edge position of the turning tool has a great effect on the machining quality, it has not been adequately explored in the existing literature.

This study examines how deflection values vary concerning the deviation of the main cutting edge from the tool holder axis using various methods and compares the results. In contrast to the commonly used deviation equation, this research applies the Castigliano theorem, which provides results much closer to actual tool deflection values. The outcomes derived from the Castigliano theorem are then compared with finite element analysis (FEM) and experimental studies. The proposed theoretical approach aligns well with the experimental results, highlighting the significance of considering torsional moments alongside bending moments as they both play a substantial role in tool deflection.

1. Introduction

Machining methods are finishing processes applied to workpieces to provide machining quality. Among these methods, the turning method is very important, and it is widely used. The dimensional quality/tolerances, geometric quality, and surface quality of the workpiece are very important in the turning process, just as in other machining methods (Basa et al. (2015)). One of the most important parameters affecting dimensional and geometric tolerances is cutting tool position, and tolerances can be adversely affected due to cutting tool deflection. In turning operations, it is very important to provide the desired geometric quality and dimensional quality in the workpiece. Cutting tool deflection is very important in the turning process, especially in geometric tolerances such as cylindricality and roundness (Lin et al. (2021)). This situation increases the importance of effective parameters during machining (Kassab and Khoshnaw (2007)), (Haddadi et al. (2008)), (Taşdemir et al., (2008)), (Lu (2008)). As seen in Fig. 1, the forces generated during the turning operation can be divided into three components: feed force (F_f), radial force (F_r), and tangential force (F_c).

In the turning process, the most important force affecting the deflection of the turning tool is the tangential force (F_c). Tangential force (F_c) is denoted as F in the tool deflection equation (Equation (1)) in the literature. In the turning process, one of the factors affecting the machining quality of the workpiece is the tool overhang (Kassab and Khoshnaw (2007)), (Clancy and Shin (2002)), (Amin et al. (2006)), (Audy (2006)), (Abouelatta and Madl (2001)), (Chiou and Liang (1998)). In turning operations, as seen in Fig. 2, the main cutting edge is displaced due to the tangential cutting force on the cutting tool.

In general, it is preferred and recommended that the tool overhang distance be as minimal as possible. However, depending on the workpiece geometry, the tool overhang may sometimes need to be as long as possible. Especially in internal turning operations, the tool holder should be attached as long as possible. In the literature, the deflection of the cutting tool is calculated assuming that the tool is a cantilever beam,

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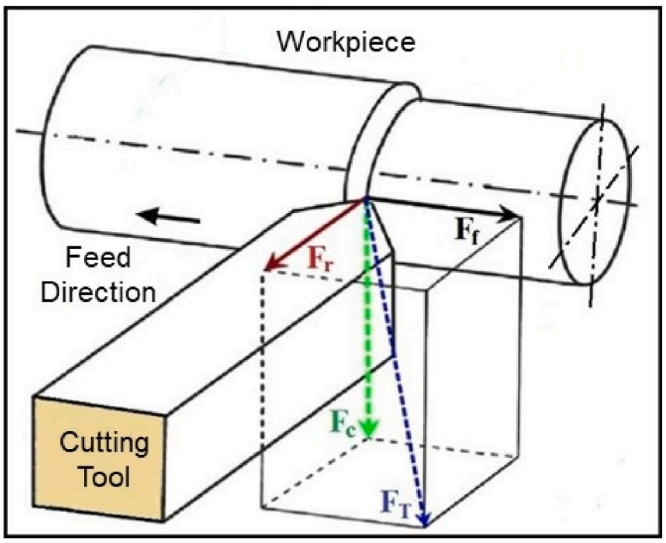


Fig. 1. Modeling of cutting forces in external cylindrical turning.

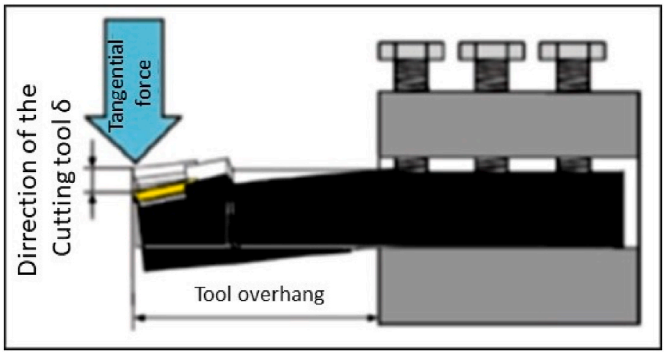


Fig. 2. Deflection of the cutting tool under the effect of cutting force.

and the cutting force is on the axis of symmetry (Fig. 3(a).

In the model shown in Fig. 3(a), only the bending moment (M_e) will occur in the tool holder as the cutting-force acts on the axis of symmetry. In this case, the commonly used tool deflection equation is as follows (Beer et al. (2008)).

$$\delta = -\frac{F.L^3}{3.E.I} \quad (1)$$

Depending on the tangential cutting-force (F), the tool overhang (L), the moment of inertia (I), and the modulus of elasticity (E), the cutting tool deflection is a function of F, L, I, and E according to Equation (1).

When the (standard) turning tools used today are examined, there are tool holders in international (ISO) standards (such as PSDNN, SRDCN, SSDCN etc.) that are geometrically similar to the cutting tool shown in Fig. 3(a).

However, other commonly used standard tool holders are geometrically different, and such tool holders are schematically given in Fig. 3 (b). In Fig. 3(b), some commonly used turning tools (and cutting force impact points relative to tool axes) are schematically modeled.

In determining the deflection value, the valid criterion for creating Equation (1) is that the effect point of the cutting force is on the symmetry axis of the cutting tool. It is seen that this assumption does not give accurate results in the deflection calculations of the cutting tool. In real conditions, the deflection value of a turning tool under the effect of cutting force will depend on the distance of the cutting force effect point from the tool axis as well as the tool overhang, as seen in Fig. 4.

The cutting tool will be displaced much more, not only due to bending but also due to the torsion effect, because the cutting edge is not on the tool axis of symmetry. With the theoretical approach given in Equation (1) in turning operations, the desired geometric quality and dimensional quality in the workpiece will not be achieved. Because the

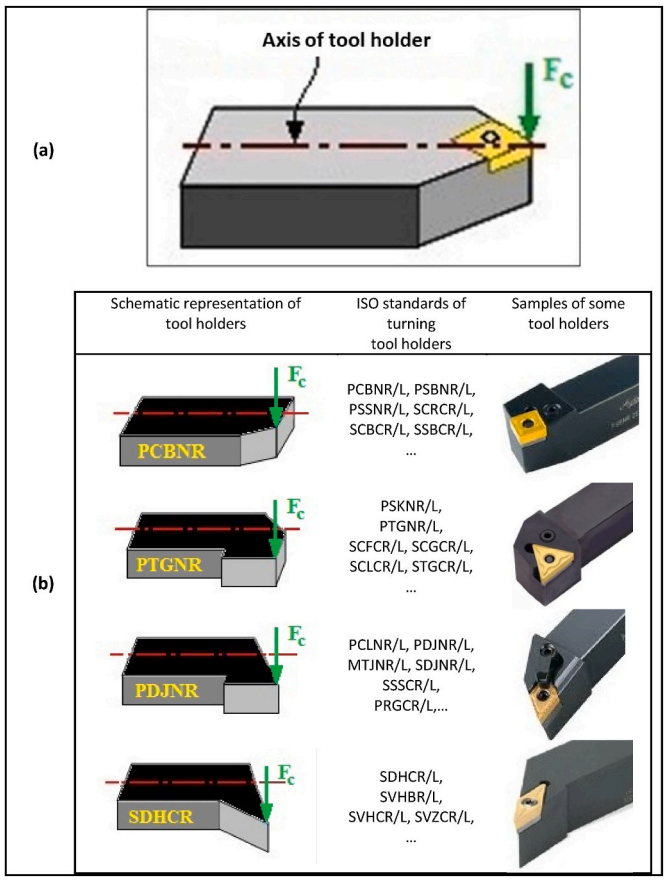


Fig. 3. a) Tool holder modeling where the cutting force acts on the symmetry axis
b) Modeling of the tool axis in some standard tool holders and the point of effect of the tangential force.

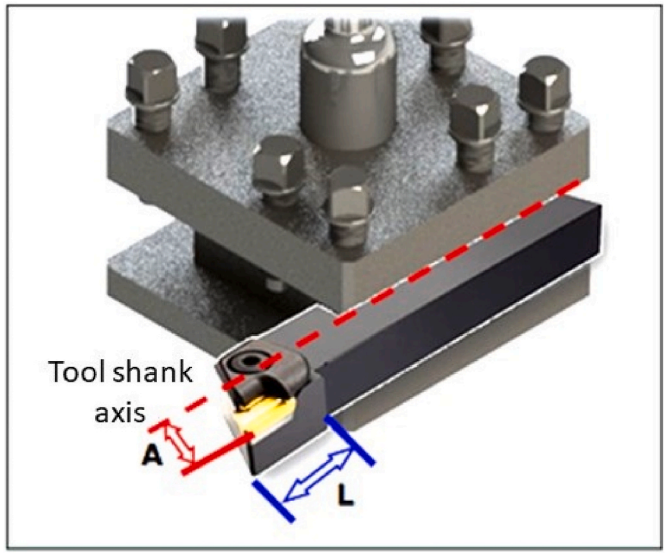


Fig. 4. Modeling for the real deflection value of the turning tool
L: Tool overhang, A: Deviation from the tool holder principal axis.

tool will be displaced more by the effect of not only bending but also torsion because the cutting edge is not in the tool symmetry axis (Kiyak et al. (2010)). Since the cutting tool is exposed to the torsional moment as well as the bending moment, much more deflection occurs at the cutting edge of the tool (Kiyak et al. (2010)). As shown schematically in Fig. 5, tool deflection will occur with the effect of both bending and torsional moments. In this case, the deflection will be much more than the deflection caused by the bending effect calculated by Equation (1).

Due to the deflection of the cutting tool, the expected machining quality cannot be achieved in the workpiece, and there may be changes in tool wear and tool life due to the change in the distribution and amount of cutting forces. Therefore, to determine the more realistic tool deflection value, it is more appropriate to apply the Castigliano theorem, which considers the effect of the torsional moment instead of only the bending moment. In this study, instead of the method that only takes into account the bending moment in the tool deflection calculation, the method that considers both the bending moment and the torsional moment is proposed and compared.

2. Applying of the Castigliano theorem to single point cutting tool

Since the position of the cutting edge is not usually on the symmetry axis of the tool holder in single-point cutting tools, the deflection Equation (1) given above does not give accurate results. The tool holder is exposed to the effect of the torsional moment as well as the bending moment. Therefore, deflection will occur on the cutting edge of the tool because of both bending and torsional moments. In this case, the cutting tool deflection will be much more than the deflection caused by the bending effect considered in Equation (1). The Castigliano theorem allows the use of strain energies at the positions of forces to determine deflections. The theorem also allows for the determination of deflections for objects with different cross-sectional areas. In an elastic system, the derivative of the strain energy concerning any external force is equal to the displacement in the direction of the force at the point where the force acts (Castigliano theorem) (Beer et al. (2008)). Castigliano theorem is given as:

$$\delta = \frac{\partial U}{\partial F} \tag{2}$$

where δ is the deflection, U is the strain energy and F is the force (or torque) at a certain point. For a cantilever beam bending.

$$U = \int_0^L \frac{M^2\, dx}{2E\, I} \tag{2.a}$$

where M is the applied moment, and I is the cross-sectional moment of inertia. For a cantilever beam in torsion;

Where T is the applied torque, G is the modulus of rigidity and J is the polar moment of inertia. If the strain energy is integrated for bending and torsion, the deflection equation is obtained as:

$$\delta = \int \frac{M_e \frac{\partial M_e}{\partial F} dx}{EI} + \int \frac{M_b \frac{\partial M_b}{\partial F} dx}{GI_p} \tag{3}$$

While the commonly used method (Equation (1)) ignores the torsional moment, the method proposed in this study (Equation (3)) considers both the bending moment and the torsional moment effect. In tool deflection calculation, Equation (3) will give much more accurate results.

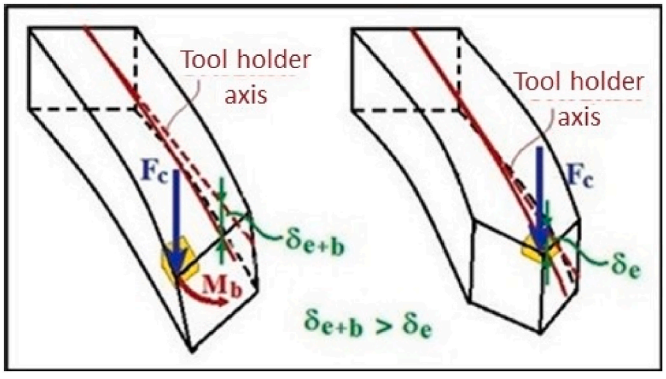


Fig. 5. Tool deflection is when the cutting force is in the tool axis and deviates from the tool axis.

2.1. Modeling of cutting tool holder type PCBNR

For this purpose, the tool holder, shown schematically in Fig. 6, was modeled and analyzed according to Equation (3).

For the modeling given in Fig. 6, Equation (4) was obtained by considering both the bending moment and the torsion moment effect.

$$\delta = \frac{FL^3}{3EI} + \frac{FL\left(\frac{a}{2} - k\right)^2}{GI_P} \tag{4}$$

when limit values are applied to Equation (4), Equation (5) below is obtained. Limit values; a: tool holder width, k is half the depth of cut. If Equation (4) is arranged by applying a: 25 mm and k = 0.5 mm, Equation (5) is obtained.

$$\delta = \frac{FL^3}{3EI} + \frac{144FL}{GI_P} \tag{5}$$

Since the cutting force, F is not in the tool holder symmetry axis, a deflection difference occurs according to Equation (1) due to the torsion moment effect. For the model in Fig. 6, the deflection difference due to the torsional moment effect is 3.268×10^{-7}xFxL. As the values of F and L increase, the calculated deflection value also increases. E.g; For L = 90 mm, F = 315 N, the deflection difference will be 9.265×10^{-3} mm. In this case, using only Equation (1) for the tool deflection calculation means ignoring a deflection difference of 8.2%.

2.2. Modeling of cutting tool holder type PDJNR

The cutting tool, modeled in Fig. 7, is accepted as two prismatic objects with dimensions of 25 × 30 × 7 and 25 × 25xL.

If the cutting force (F) acts in the middle of the depth of the cut, the value k is taken as half the depth of the cut. For this standard cutting tool, the following Equation (6) is obtained, taking into account the effect of both bending moment and torsion moment, depending on the modeling.

$$\delta = \int_0^7 \frac{M_e \frac{\partial M_e}{\partial F} dx}{EI_1} + \int_0^7 \frac{M_b \frac{\partial M_b}{\partial F} dx}{GI_{p_1}} + \int_0^{L-15} \frac{M_e \frac{\partial M_e}{\partial F} dx}{EI_2} + \int_0^{L-15} \frac{M_b \frac{\partial M_b}{\partial F} dx}{GI_{p_2}} \tag{6}$$

If the bending moment (M_e) and torsional moment (M_b) values are arranged by substituting them in Equation (3), the following result is obtained. The tool deflection is obtained as a function of F, k, and L in Equation (6.a).

$$\delta = P.F + Q.F(15-k)^2 + \frac{F(L-15)^3}{R} + \frac{F(15-k)(L-15)^2}{S} + \frac{F(15-k)^2(L-15)}{S} + T.F(L-15) \tag{6.a}$$

P, Q, R, S, T coefficients are scalar coefficients containing the values G, E, IP_1, IP_2, I_1, I_2. Contrary to Equation (1), which is widely used in the literature, in the turning process, a more realistic approach is provided by considering the effect of the torsion moment on tool deflection. For example, for F = 315 N, k = 0.5, and L = 90 mm, there is a 19% difference when Equation (1) and Equation (6) are compared. Thus,

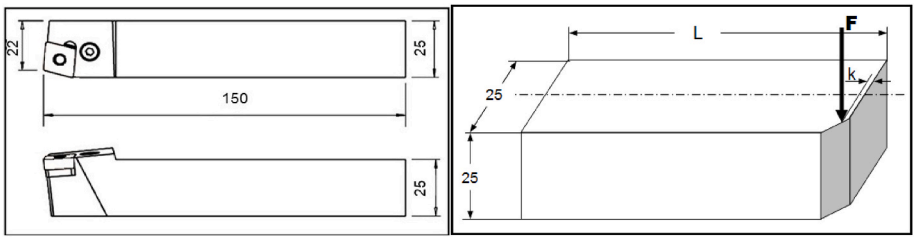


Fig. 6. Modeling of PCBNR-type cutting tool holder.

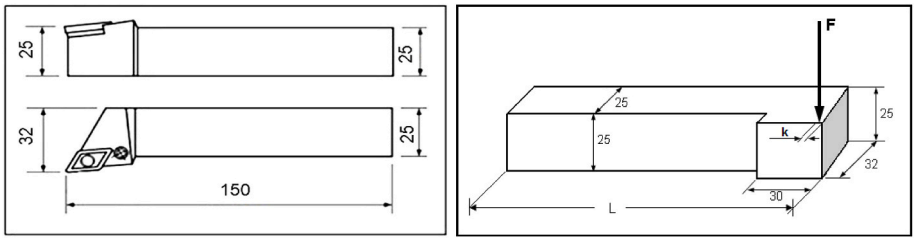


Fig. 7. Modeling of the cutting tool holder (PDJNR).

considering the effect of the torsion moment, a more realistic approach is provided compared to Equation (1).

3. FEM analyses for real tool deflections

One of the most accurate numerical solution methods for modeling cutting processes is the finite element method (FEM), which is widely used today. In this section, actual tool deflection values were determined using FEM. The deviation of the cutting tools under the influence of cutting forces was determined using the ANSYS® finite element program. Simulation conditions for FEA analysis are; steel tool holder, WC insert and 500 N load.

The cutting tool material is assumed to have linear elastic isotropic properties. C3D4 was chosen as the element type. Thus, a solid element with a 3-dimensional 4-node tetrahedron structure with linear displacement was used. A uniform mesh is used in the cutting tool. The resulting values obtained using FEM (ANSYS®) are shown in Fig. 8.

It is seen in the results that the deflection values increase with the increase in the distance between the tool cutting edge and the axis of the cutting tool holder. The results of the FEM analysis values obtained according to the tool holder types are given in Table 1.

It is seen that the obtained FEM results are quite compatible with the literature (Kiyak (2020), (Gasagara et al. (2019)), (Renjith et al. (2013)). As seen in Tables 1 and if the cantilever beam deflection approach (Equation (1)) is applied, the displacement will give the same values (0.0022 mm) for all tool holder types. The deflection values increase when the torsional effect is taken into account. When only Equation (1) is used, the tool deflection values for different types of cutting tools are the same (0.0022 mm). However, when using the deflection Equation (Equation (3)), which takes into account the effects of both bending and torsional moments, the actual deflection values are 0.0044 mm and 0.0154 mm, respectively.

Table 1
Comparison of Equation (1) and Equation (3) with FEM analysis and experimental results.

	PCBNR type tool holder	PDJNR type tool holder
Deflection of cutting tool holder (according to Equation (1))	0,0022 mm	0,0022 mm
Deflection of cutting tool holder (Bending moment + Torsion moment) (Equation (3))	0,0044 mm	0,0154 mm
Results of FEM models	0,0136 mm	0,0189 mm

3.1. Experimental verification

Depending on the position of the cutting edge of the tool holder, the actual deflection values that will occur with the effect of both the bending moment and the torsional moment were determined by the experimental study shown in Fig. 9. Mercer brand 122 L model electronic comparator was used to determine the deflection values precisely. The tool holders modeled in Figs. 6 and 7 are attached to the tool holder on the lathe.

To determine the deflection of the cutting tool in real conditions, calibration weights were hung on the tool holder and deflection values were measured with an electronic comparator. As a result of the experimental studies, the actual deflection values in two different tool holders depending on the tool overhang are given in Fig. 10.

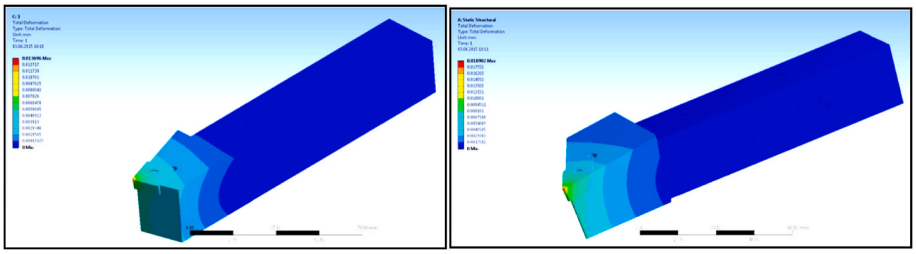


Fig. 8. FEM Analysis results for PCBNR and PDJNR type tool holders.

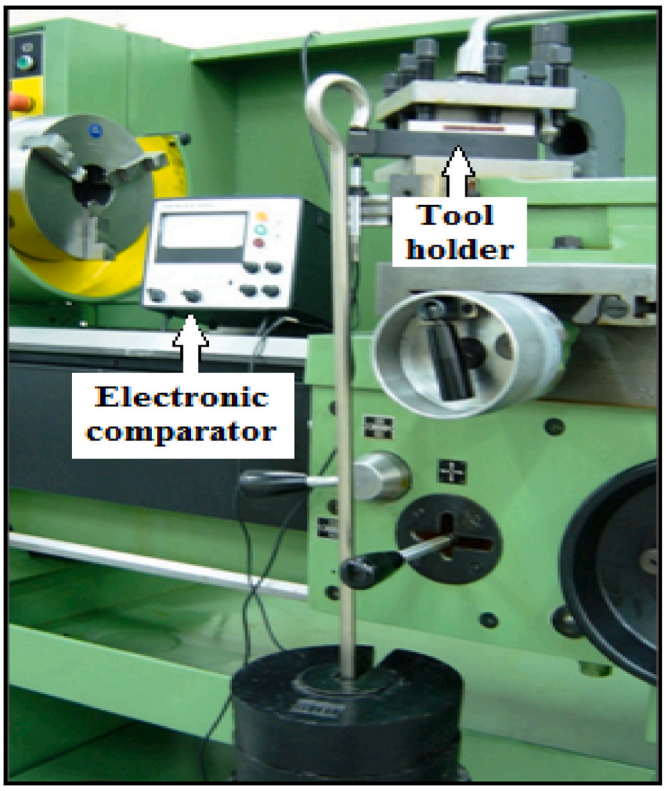


Fig. 9. Experiment set in which the actual deflection value of the cutting tool is measured.

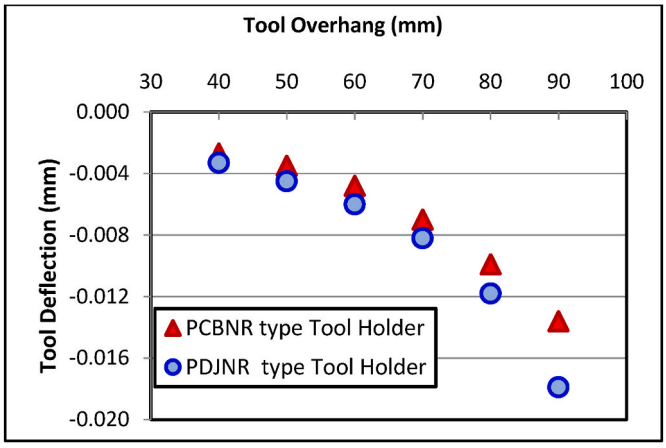


Fig. 10. Actual deflection values of different tool holders depending on the tool overhang.

4. Results and discussion

It has been demonstrated by this study that a more accurate result will be achieved by taking into account the torsional moment effect of the deflection that will occur in the single-point cutting tool during turning. The deflection values that occur depending on the cutting edge position in a single-point cutting tool are given in Fig. 11 comparatively. The curve indicated in red color in Fig. 11 is the result of the experimental study using a PCBNR-type tool holder. The blue colored curve is the experimental deflection value of the PDJNR-type cutting tool holder. The closest curve to the blue and red colored curves obtained from the experimental results is Equation (3) curve, which includes both bending and torsional moments. The green-colored curve was obtained from the

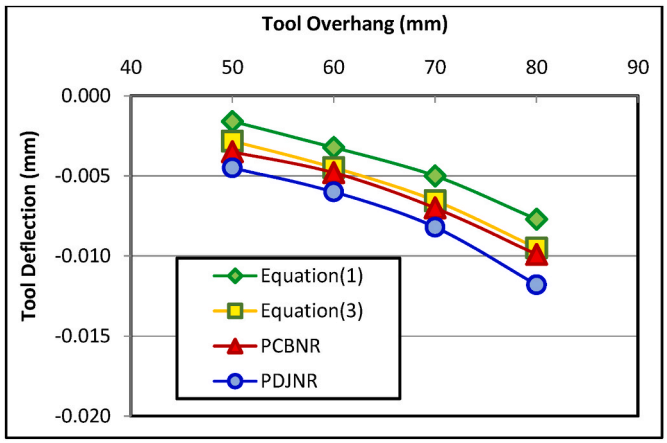


Fig. 11. Comparison of analytical and experimental deflection values.

deflection equation (Equation (1)), which is widely used in the literature. As can be seen, the results obtained from Equation (1) are quite far from the actual deflection values.

Equation (1), which is widely used in the literature to determine tool deflection, is independent of the distance between the main cutting edge and the tool holder axis. This distance is shown as "A" in Fig. 4. If the tool cutting edge is not on the tool axis, in addition to the bending moment, a torsional moment will also occur.

Under the influence of both moments, the cutting tool deflection will be much higher. In Equation (3), both the bending moment and torsional moment are considered. In the experiments carried out with turning tools in ISO standards, it was observed that the deflection values increased as the distance between the main cutting edge and the tool holder axis increased. In Fig. 12, the differences between the results of Equation (1) and Equation (3) are given graphically.

Equation (1) always gives the same result independently of the tool holder type. The results are shown with the blue colored curve in Fig. 12. However, the cutting tool deflection increases depending on the distance "A". The cutting tool deflection difference between Equation (1) and Equation (3) is 8.2% for the PCBNR type tool holder, 19% for the PDJNR tool holder and 30.9% for the SCHCR type tool. FEM analysis for different types of tool holders also supports Equation (3). If Equation (1) is used, the cutting tool deflection for each type of cutting tool holder is 0.0022 mm. Comparisons of Equation (1) and FEM analyses are given in Fig. 13.

In the FEM analyses given in Fig. 8, the cutting tool deflections were found to be 0.0136 mm for the PCBNR tool holder, 0.0189 mm for the PDJNR tool holder, and 0.0208 mm for the SCHCR tool holder.

These values obtained by FEM analysis are in good agreement with

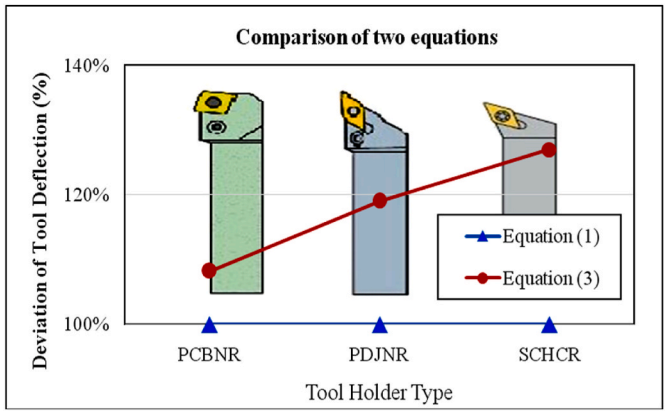


Fig. 12. The amount of deviation in % between the two equations.

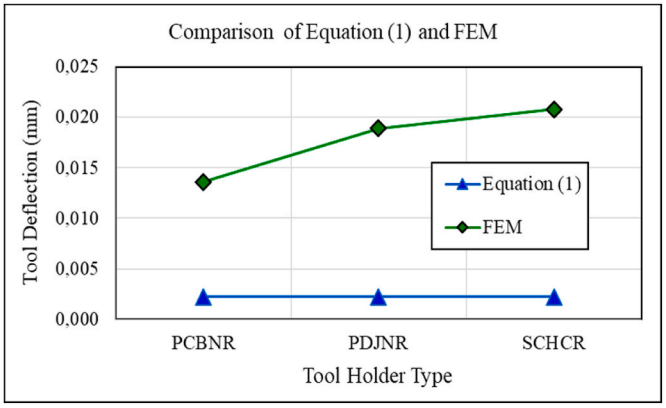


Fig. 13. Comparison of Equation (1) and FEM results.

the experimental results (red and blue colored curves) obtained as a result of the experiments and given in Fig. 11. As a result, when compared with the results obtained from the experimental data, it was seen that Equation (3) obtained by the Castigliano theorem provides a much better approximation than Equation (1).

5. Conclusion

In this study, the actual tool deflection value according to the cutting-edge position for tool holders in ISO standards has been tried to be revealed. The important variables in this study are the distance of the main cutting edge of the tool from the tool axis (distance A) and the tool overhang (L). In the study, cutting tool holders (PCBNR, PDJNR, SCHCR), which are widely used in the manufacturing industry, were selected, and examined. As a result, it is seen that Equation (3) proposed in the study gives much more accurate results than Equation (1).

In the future of this study.

- Tool deflection values for all turning tools, standard or not, can be precisely determined by taking into account the effect of cutting-edge position and torsional moment.
- More accurate tool deflection values can be determined for all standard cutting tool cross-sections (eg 10 × 10, 20 × 20, 25 × 25, 25 × 32 mm).
- This study has been done for standard external turning tools, but can also be performed for internal turning tools and boring tools.

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